Model Predictive Control for Automotive Applications

Paolo Falcone

Dipartimento di Ingegneria "Enzo Ferrari"
Universita' di Modena e Reggio Emilia

UNIMORE
UNIVERSITÀ DEGLI STUDI DI MODENA E REGGIO EMILIA

CHALMERS
UNIVERSITY OF TECHNOLOGY

Department of Electrical Engineering
Chalmers University of Technology
Göteborg, Sweden

Predictive Vehicle Dynamics Control

Problem complexity

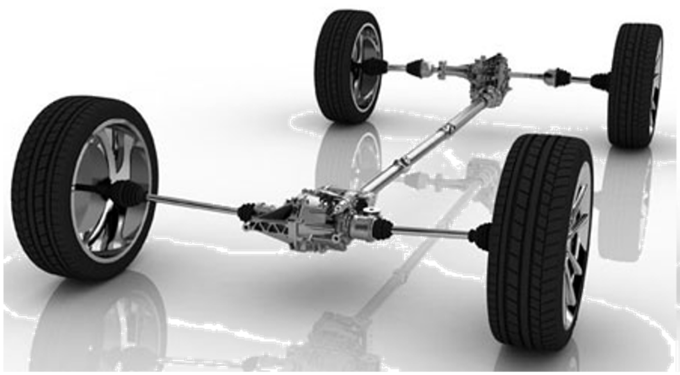
How does the logic change if further actuators are added?

Engine torque

Active front/rear steering

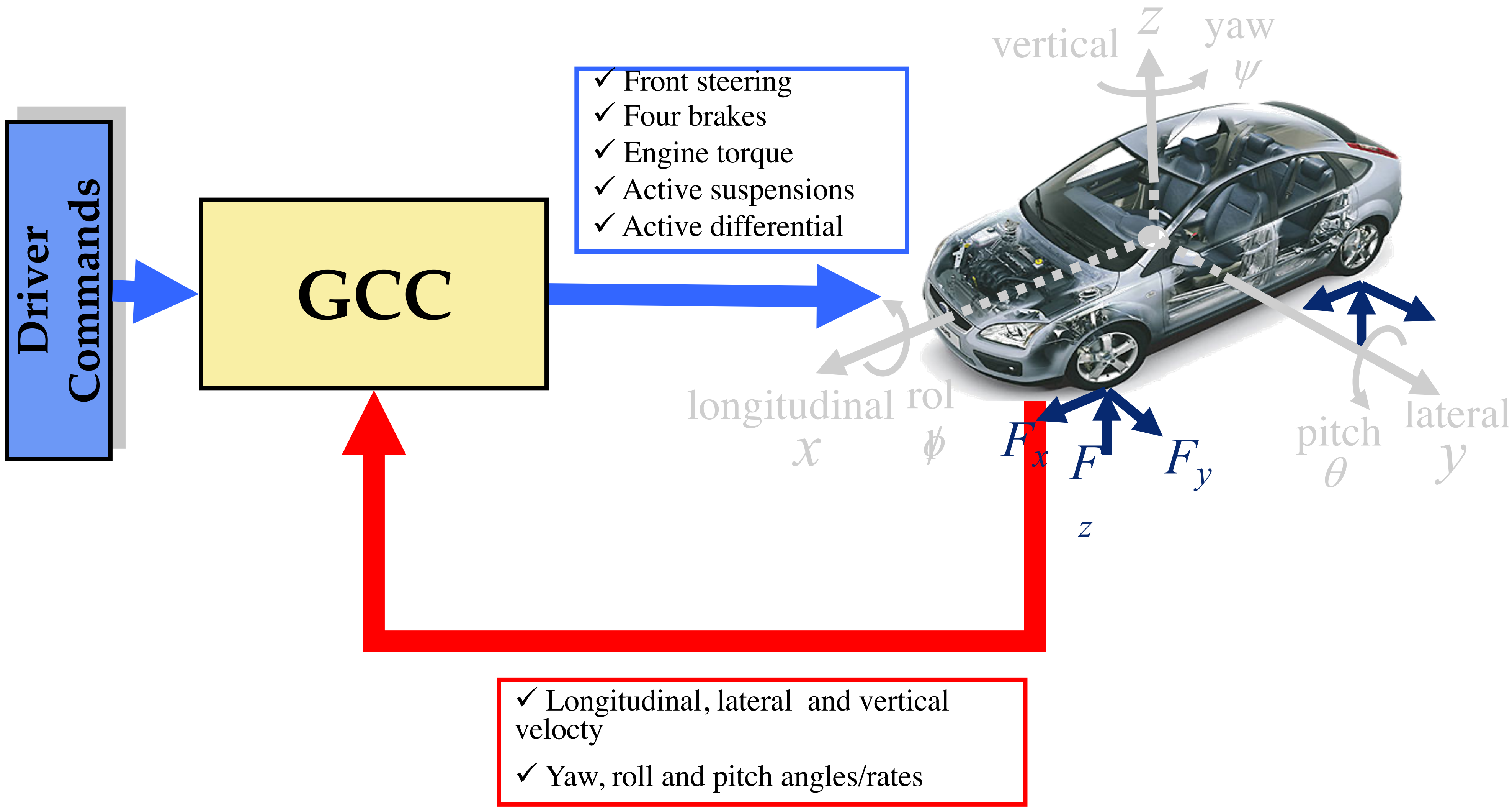
Active suspensions

Active differentials



Global Chassis Control (GCC) problem

Coordinating vehicle actuators in order to control multiple dynamics



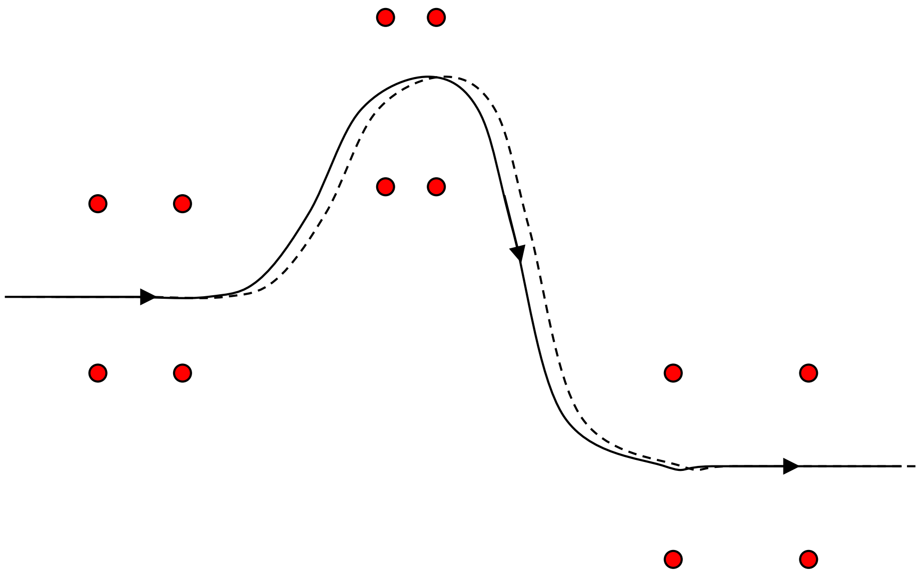
Testing scenario. Autonomous path following

Problem setup:

- Double lane change
- Driving on snow/ice, with different entry speeds

Control objective:

Minimize angle and lateral distance deviations from reference trajectory by changing the ***front wheel steering angle*** and the ***braking at the four wheels***

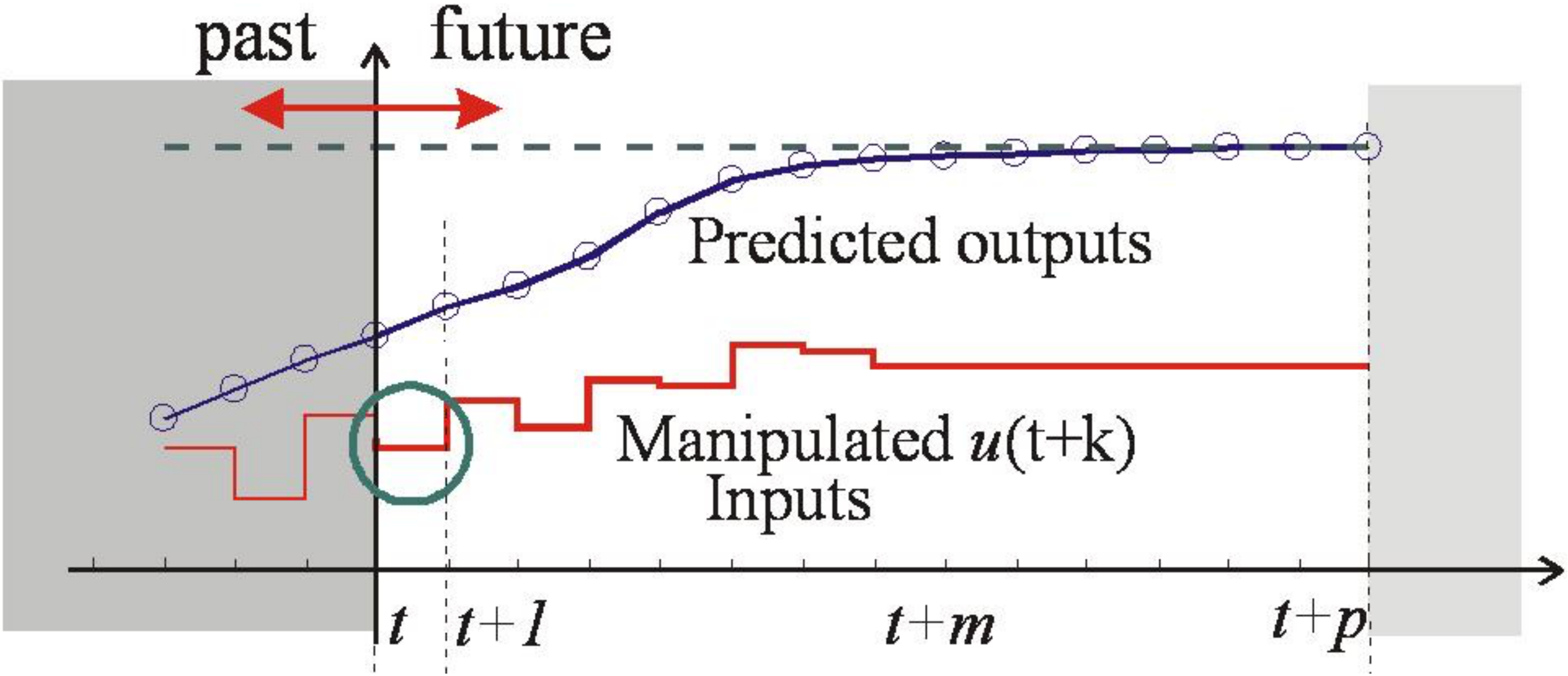


Controlling longitudinal, lateral and yaw dynamics by varying front steering angle and braking the four wheels

Challenges

- Highly nonlinear MIMO system with uncertainties
  – Tires characteristics
- 6 DOF model
  – Longitudinal, lateral, vertical, roll, yaw and pitch dynamics
- Hard constraints
  – Rate limitations in the actuators, vehicle physical limits
- Fast sampling time
  – Typically 20-50 ms

Model Predictive Control



Main ingredients

- Vehicle model
- Optimization problem

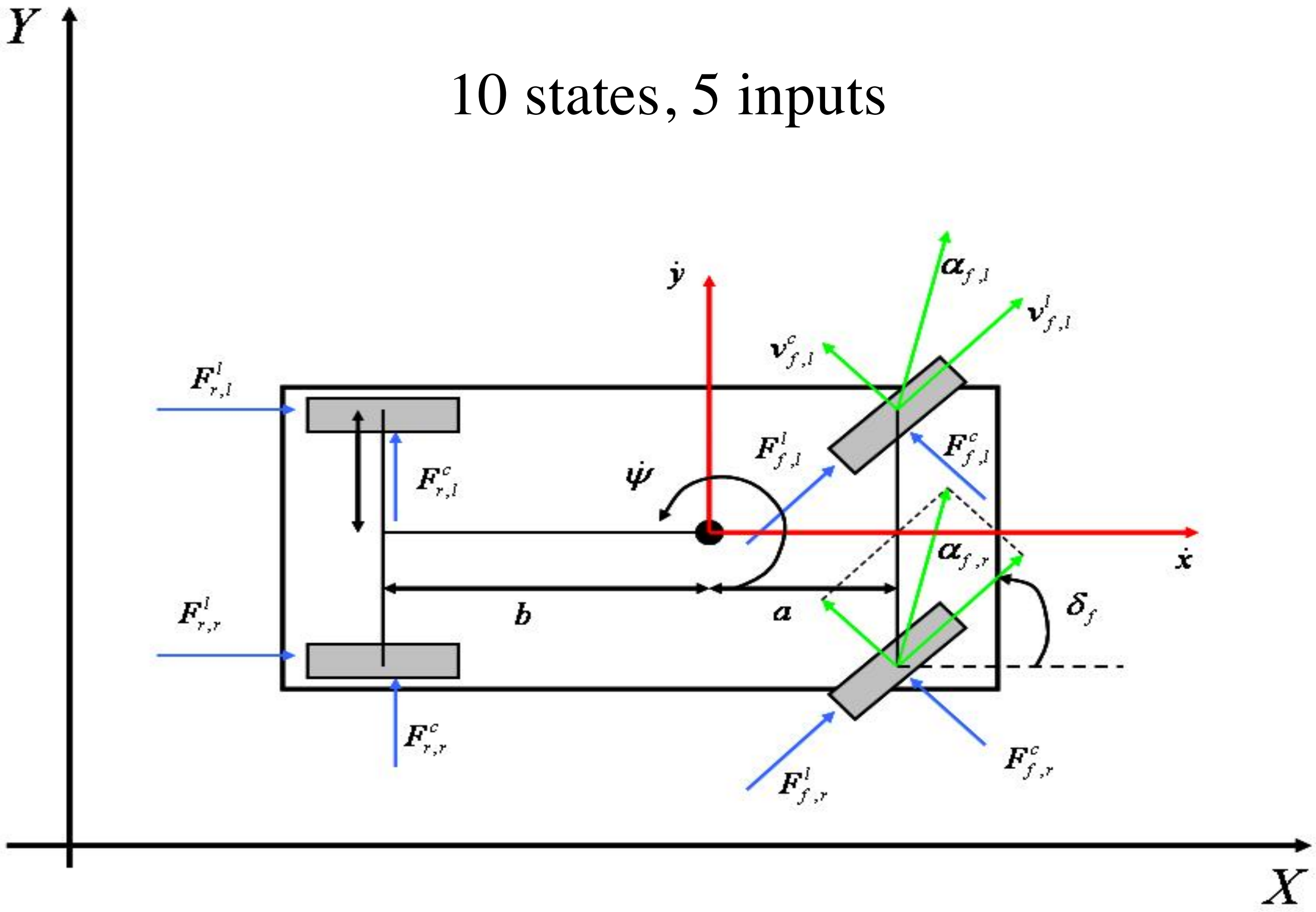
Outline

- Introduction and motivations
- Vehicle modeling
- Problem formulation
- Experimental results

Modeling: bicycle and four wheels models

Modeling the vehicle motion in an inertial frame subject to lateral, longitudinal and yaw dynamics

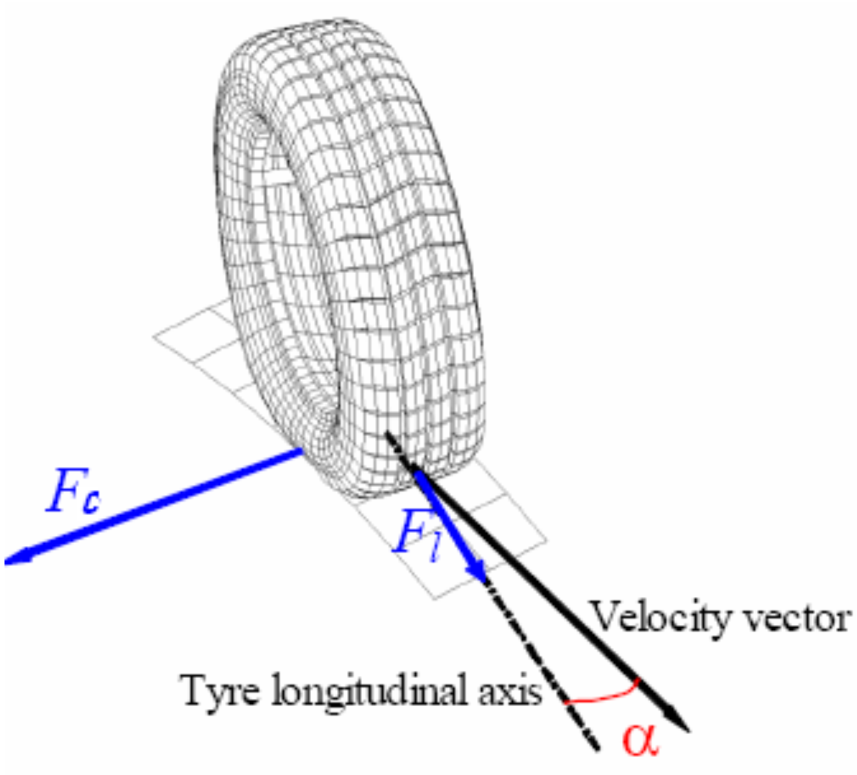
10 states, 5 inputs

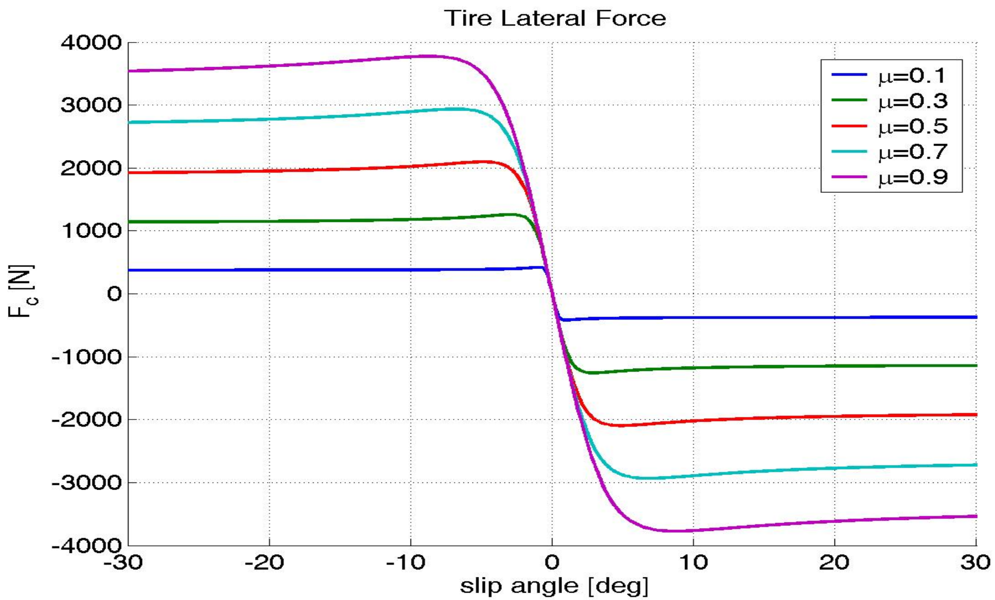


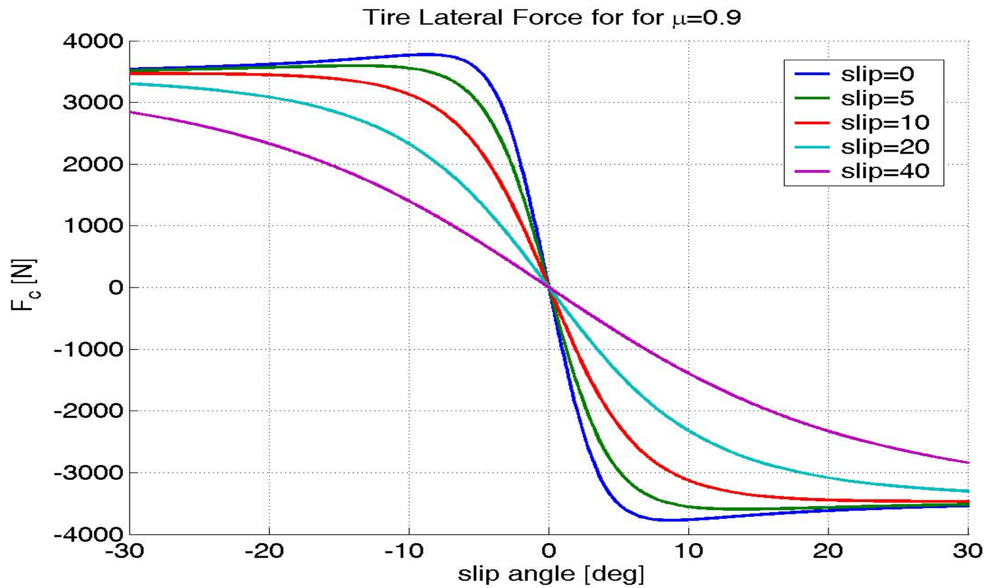
Tire modeling

Static tire forces characteristics

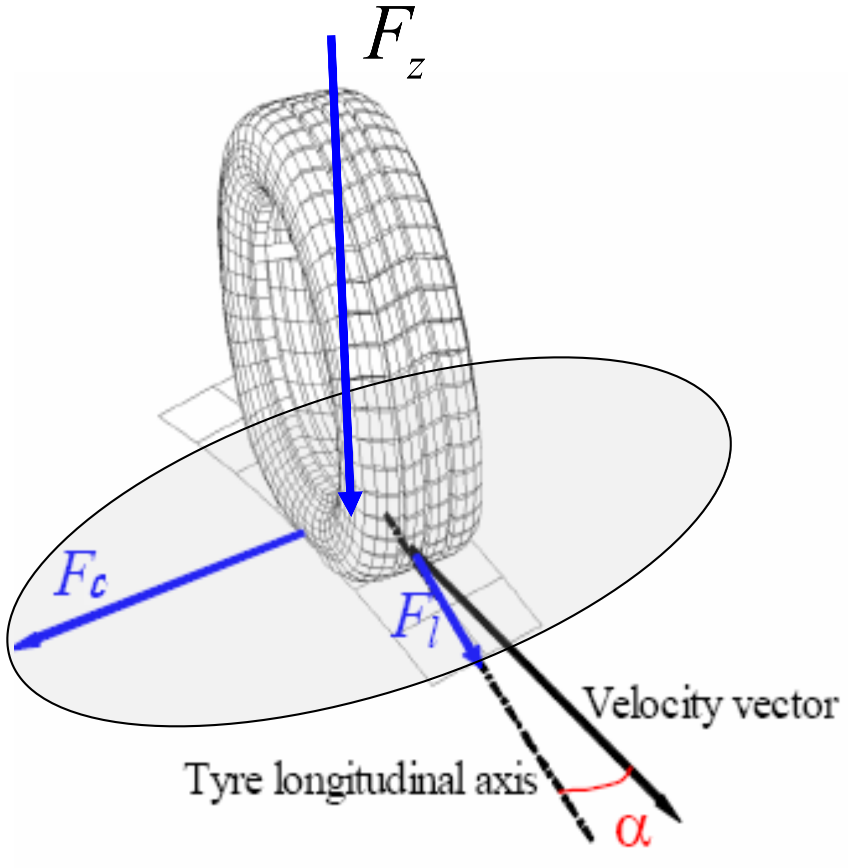
$$F_c = f_c(\alpha, s, \mu, F_z)$$

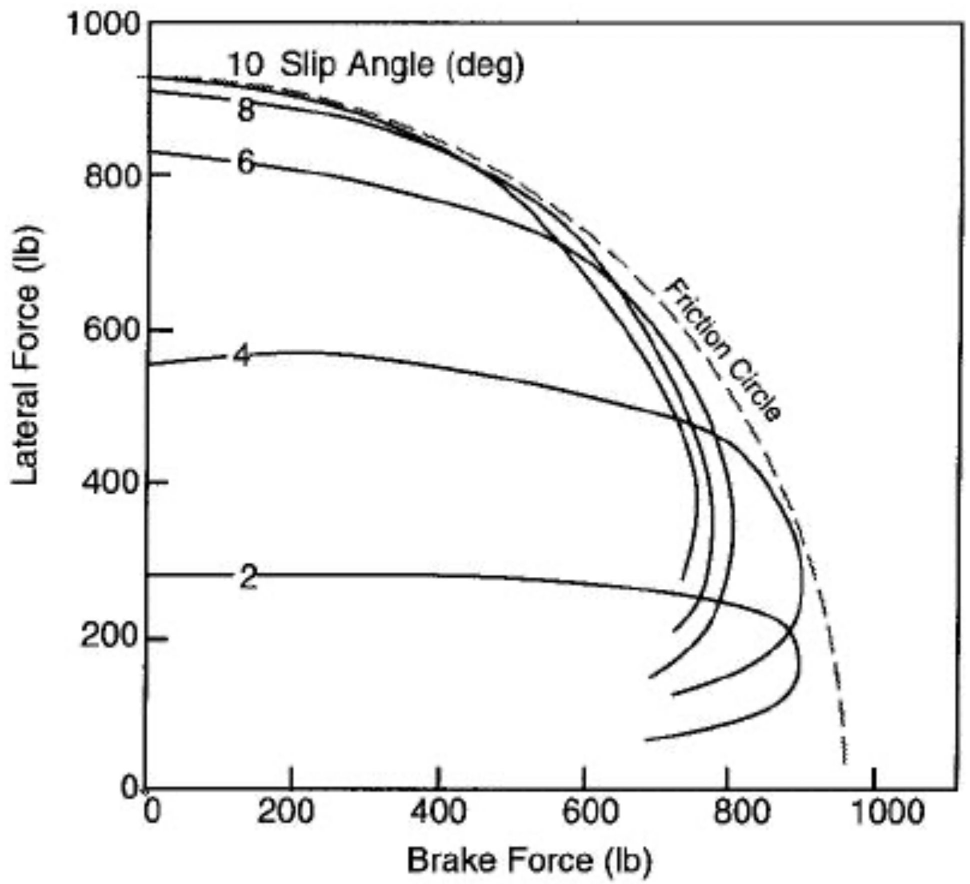






Tire modeling





$$\sqrt{F_l^2 + cF_c^2} \leq \mu F_z$$

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NMPC Control design

Cost function

$$J(\xi(t),U) = \sum_{i=1}^{H_p} \left\| \eta_{t+i,t} - \eta_{ref_{t+i,t}} \right\|_Q^2 + \left\| u_{t+i,t} \right\|_R^2 \qquad U = u_{t,t} \ldots u_{t+H_p,t}$$

Optimization problem	$\min_{U} J(\xi_t, U)$ subj. to
Vehicle dynamics	$\xi_{k+1,t} = f(\xi_{k,t}, u_{k,t})$ $\eta_{k,t} = h(\xi_{k,t})$
Input constraints	$u_{\min} \le u_{k,t} \le u_{\max}$
Constraints on input changes	$\Delta u_{\min} \le \Delta u_{k,t} \le \Delta u_{\max}$
	$k = t, \ldots, t + H_p - 1$

• Non Linear Programming (NLP) problem

• Complex NLP solvers required

Real time implementation by
✓ limiting the number of iterations
✓ using short horizons

Experimentally tested

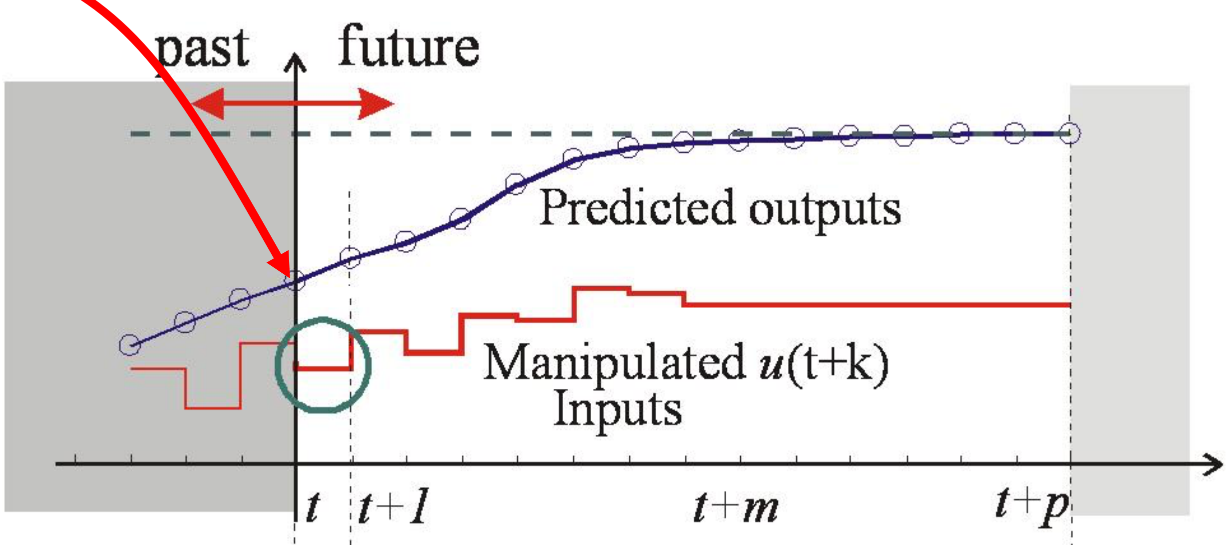
Real time testing at *low* speed

LTV-MPC controller

*Approximating the non-linear vehicle model with a Linear Time Varying (LTV) model A_t, B_t, C_t, D_t.**

* Kothare and Morari, 1995, Wan and Kothare, 2003

$$A_t, B_t, C_t, D_t$$



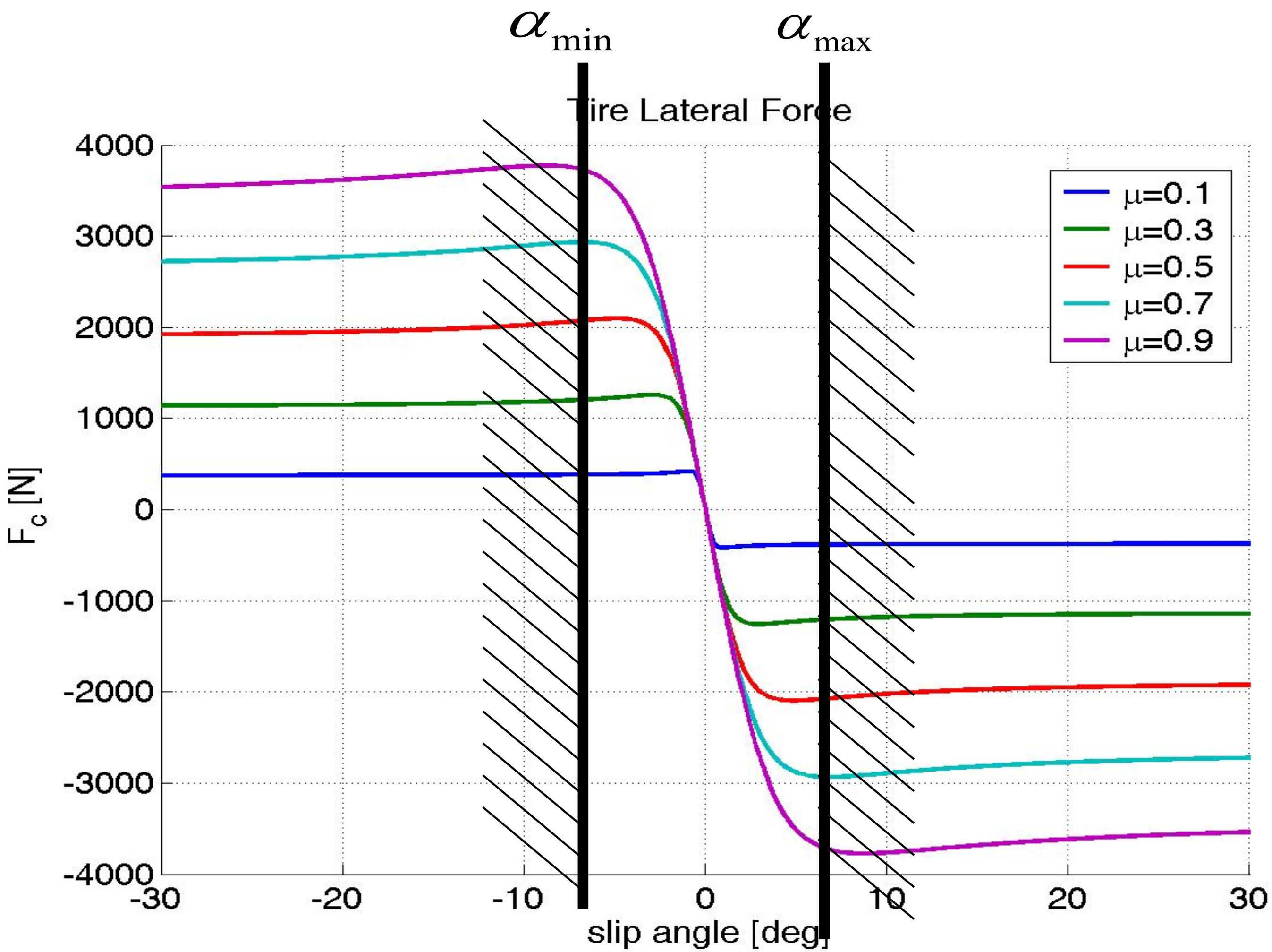
- Convex optimization problem. QP solvers
- Easier real-time implementation
- Longer horizons

Performance and ***stability*** issues* due to linear approximation

* Falcone et al 2008

Constraints on tire slip angle

Stability achieved through *ad hoc* state and input constraints



$$\alpha_{\min_{k,t}} \leq \alpha_{k,t} \leq \alpha_{\max_{k,t}}$$
$$k = t \ldots t + H_p$$

Controller performs well up to 21 m/s

The system is still nonlinear

Stability of the LTV-MPC approach

Consider the discrete time nonlinear system:

$$(1) \qquad \xi(t+1) = f\big(\xi(t), u(t)\big) \qquad \xi \in R^n \quad u \in R^m$$

We consider the following linear approximation over the horizon *N*:

$$\xi(k+1) \cong A_{k,t}\xi(k) + B_{k,t}u(k) + d_{k,t}$$
$$k = t, \dots t+N-1$$

$$A_{k,t} = \frac{\partial f}{\partial x}\Big|_{\substack{\xi_0(k) \\ u(t-1)}}, \quad B_{k,t} = \frac{\partial f}{\partial u}\Big|_{\substack{\xi_0(k) \\ u(t-1)}}$$

$$\xi_0(k+1) = f\big(\xi_0(k), u(t-1)\big), \qquad \xi_0(k) = \xi(t)$$
$$d_{k,t} = \xi_0(k+1) - A_{k,t}\xi_0(k) - B_{k,t}u(t-1)$$

Stability of the LTV-MPC approach

$$(2) \quad V_N^*\left(\xi(t)\right) = \min_{u_{t,t},\ldots u_{t+N-1,t}} \sum_{i=1}^{N-1} \left\|Q\xi_{t+i,t}\right\|_2^2 + \sum_{i=0}^{N-1} \left\|Ru_{t+i,t}\right\|_2^2 + \left\|P\xi_{t+N,t}\right\|_2^2$$

subject to:

$$\xi_{k+1,t} = A_t \xi_{k,t} + B_t u_{k,t} + d_{k,t}, \qquad k = t,\ldots,t+N-1$$

$$\xi_{k,t} \in X, \qquad k = t,\ldots,t+N-1$$

$$u_{k,t} \in U, \qquad k = t,\ldots,t+N-1$$

$$\xi_{k+N,t} \in X_f$$

$$\xi_{t,t} = \xi(t)$$

$$(3) \qquad u(t) = u_{t,t}^*\left(\xi(t)\right)$$

Stability of the LTV-MPC approach

Theorem. The system (1) with the control law (2)-(3), where $X_f=0$, is *uniformly asymptotically stable* if

$$\left\|Q\hat{\xi}_{t+N-1,t}\right\|_2^2 + \left\|Ru_{t+N-1,t}\right\|_2^2 \le \left\|Q\xi^*_{t,t-1}\right\|_2^2 + \left\|Ru^*_{t-1,t-1}\right\|_2^2 - \sum_{i=1}^{N-2}\left\|Q\left(\hat{\xi}_{t+i,t} - \xi^*_{t+i,t-1}\right)\right\|_2^2 - \gamma$$

where:

$$\hat{\xi}_{k+1,t} = A_t\hat{\xi}_{k,t} + B_t u^*_{k,t-1} + d_{k,t}$$

$$k = t,\dots,t+N-2$$

$$\hat{\xi}_{t,t} = \xi(t)$$

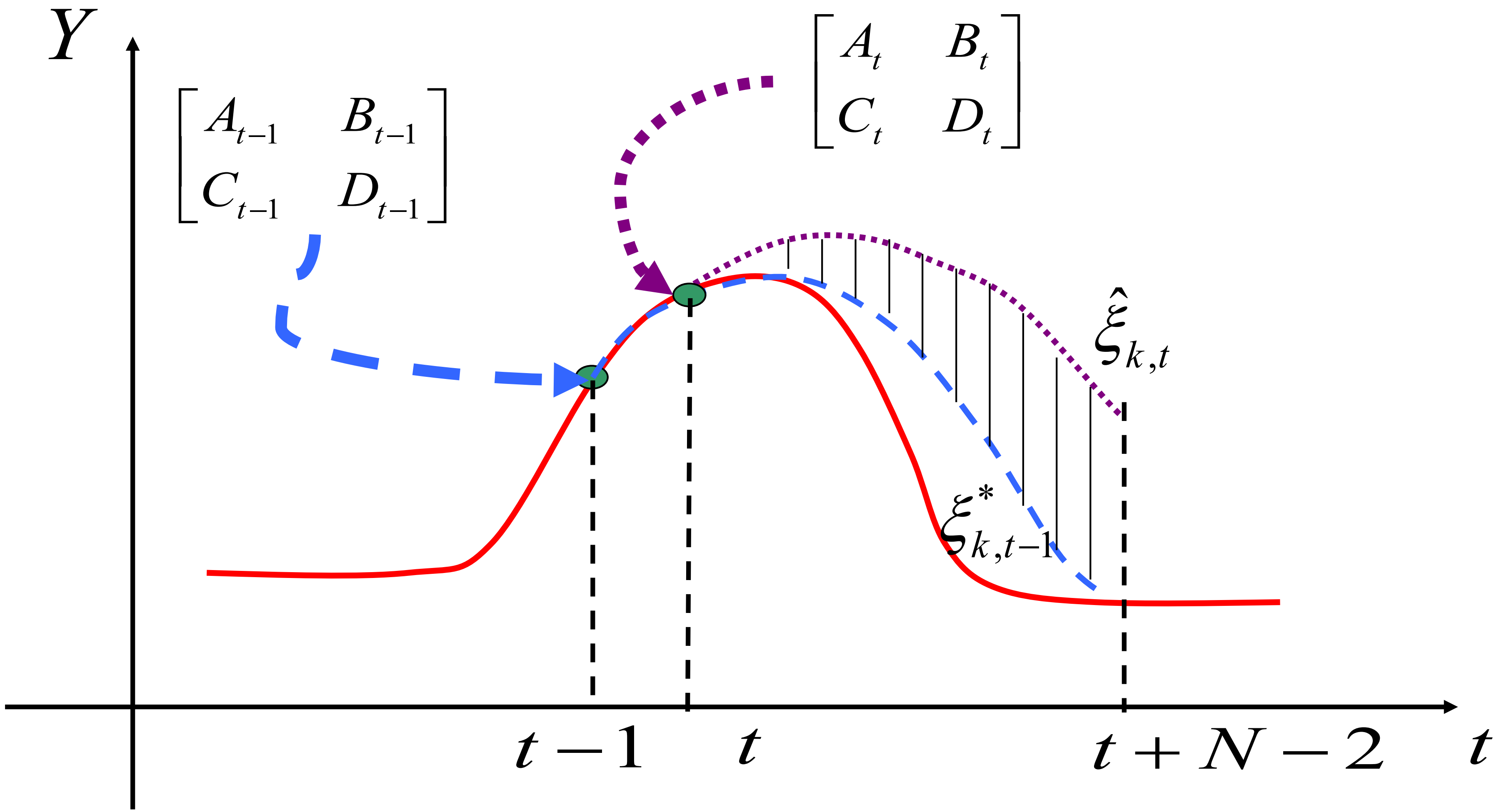
$$\gamma = 2\sum_{i=1}^{N-2}\left\|Q\left(\hat{\xi}_{t+i,t} - \xi^*_{t+i,t-1}\right)\right\|_2 \left\|Q\xi^*_{t+i,t-1}\right\|_2$$

State and input convex constraints

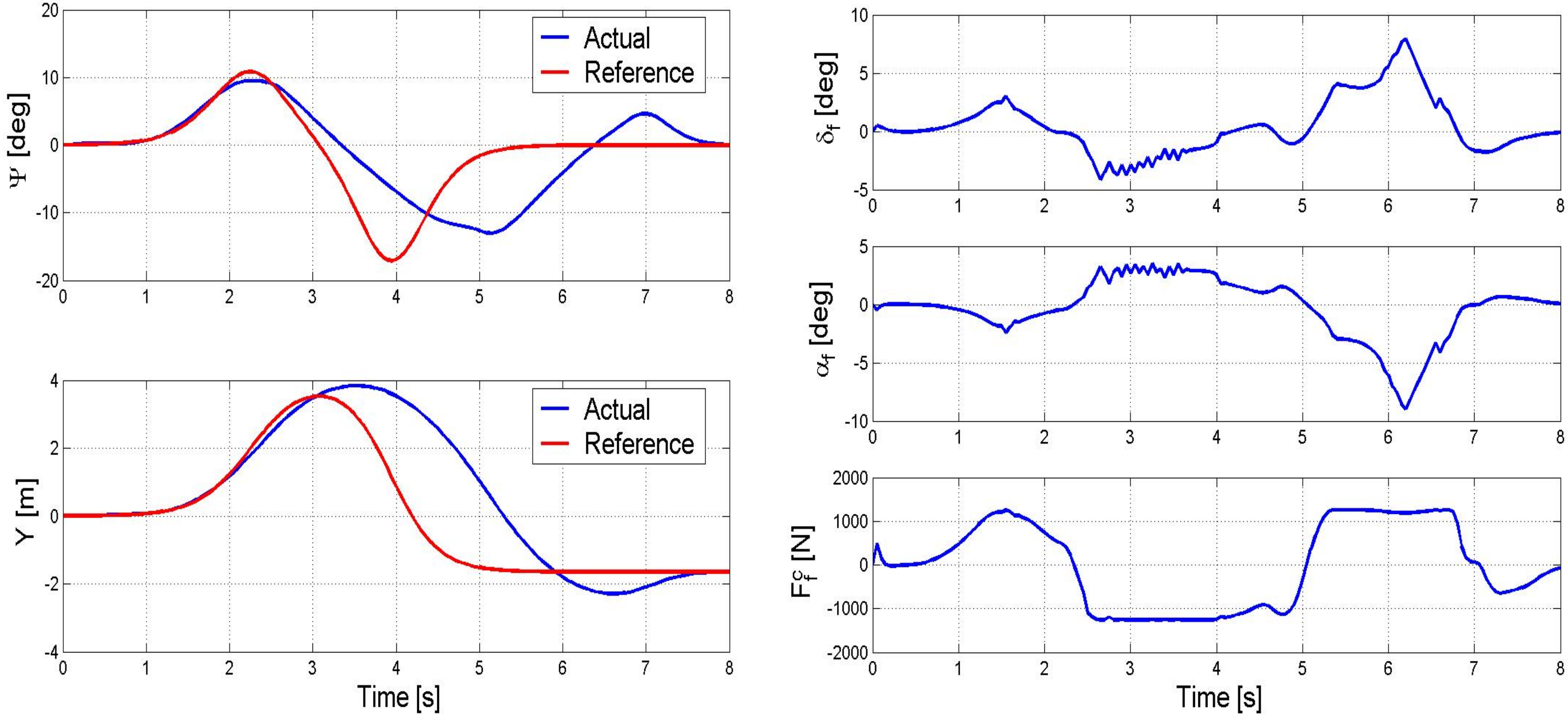
Liu, 1968. Chen and Shaw. 1982. Mayne *et al*. 2000

What does that mean?

$$l\left(\hat{\xi}_{t+N-1,t}, u_{t+N-1,t}\right) \leq l\left(\xi^*_{t-1,t-1}, u^*_{t-1,t-1}\right) - \Gamma\left(\left\| Q\left(\hat{\xi}_{t+i,t} - \xi^*_{t+i,t-1}\right)\right\|_2^2\right)$$



Simulation results at 21 m/s



The controller is able to stabilize the vehicle without any 'ad hoc' constraint.

Outline

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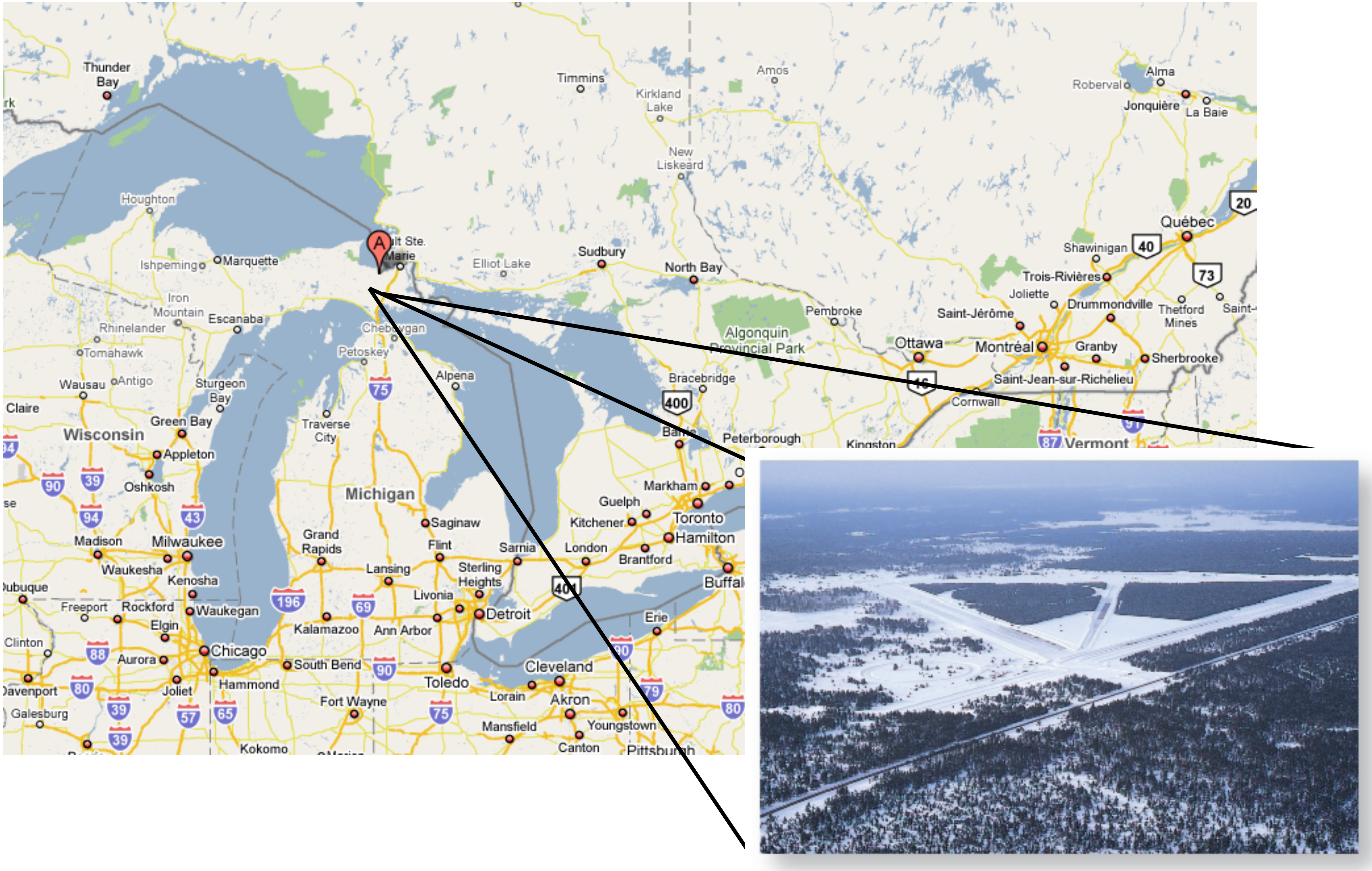
Testing: Sault St. Marie in Upper Peninsula, MI, USA



Summary

- Excellent performance
- Limited tuning effort (less than 10 run ~ 1 hr)
- Vehicle stabilized up to 70 Km/h on snowy tracks

- Coordination of steering and braking
  - Braking is delivered on the “same side” of the steering
- Front/rear braking distribution
  - Shifting the braking to the non saturated axle
- Countersteering
  - Steering in opposite direction of path following to prevent spinning

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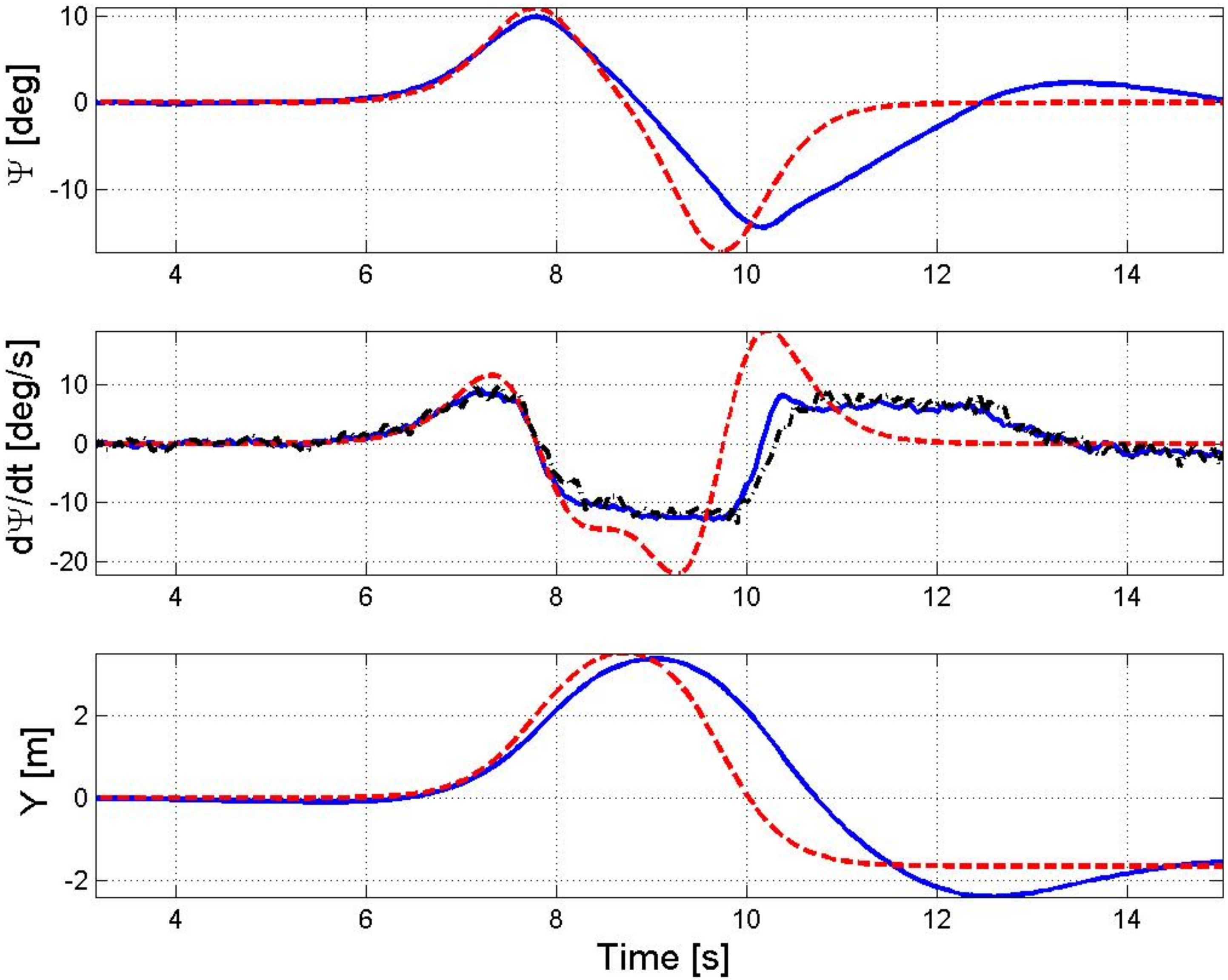
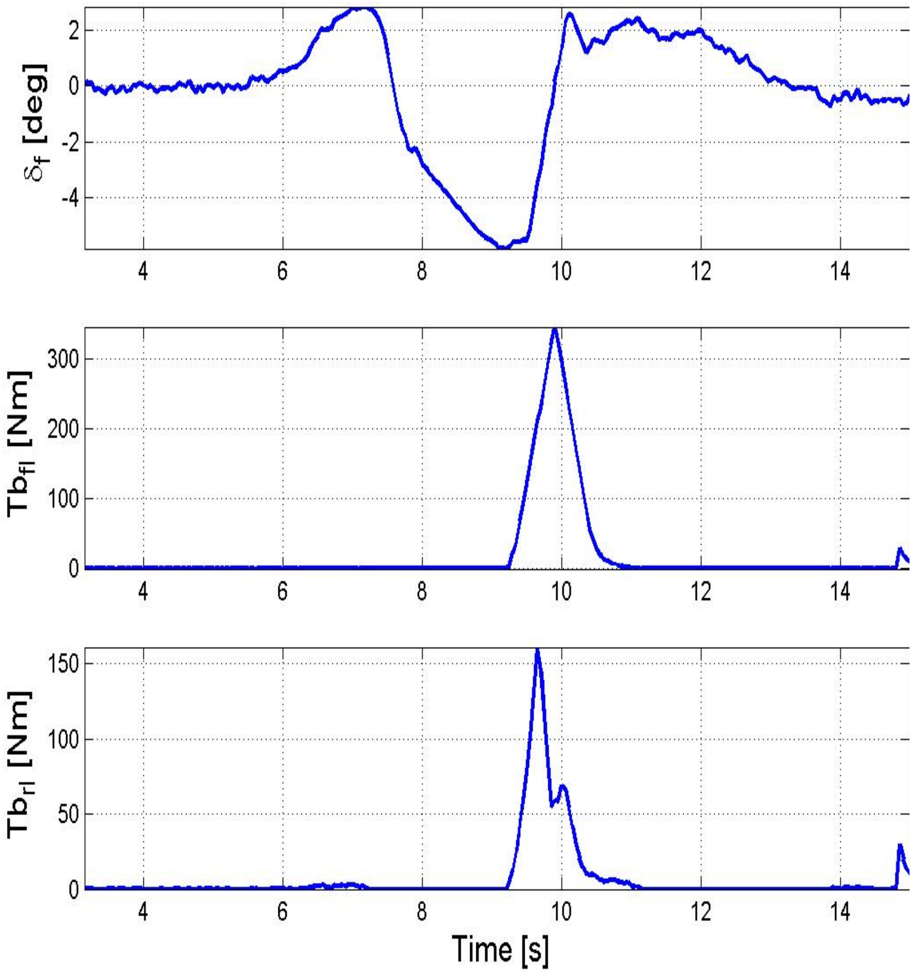
Test @ 40 Kph. Steering and braking coordination

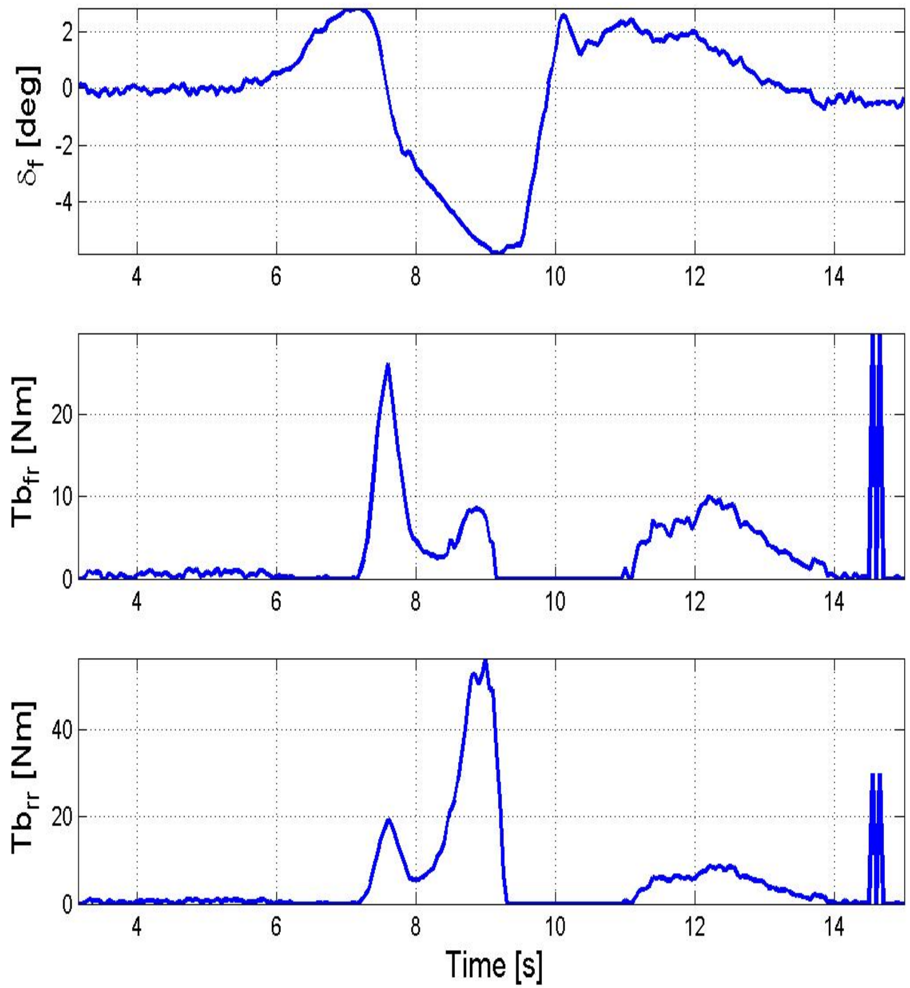


Test @ 40 Kph. Steering and braking coordination

Left side braking



Right side braking



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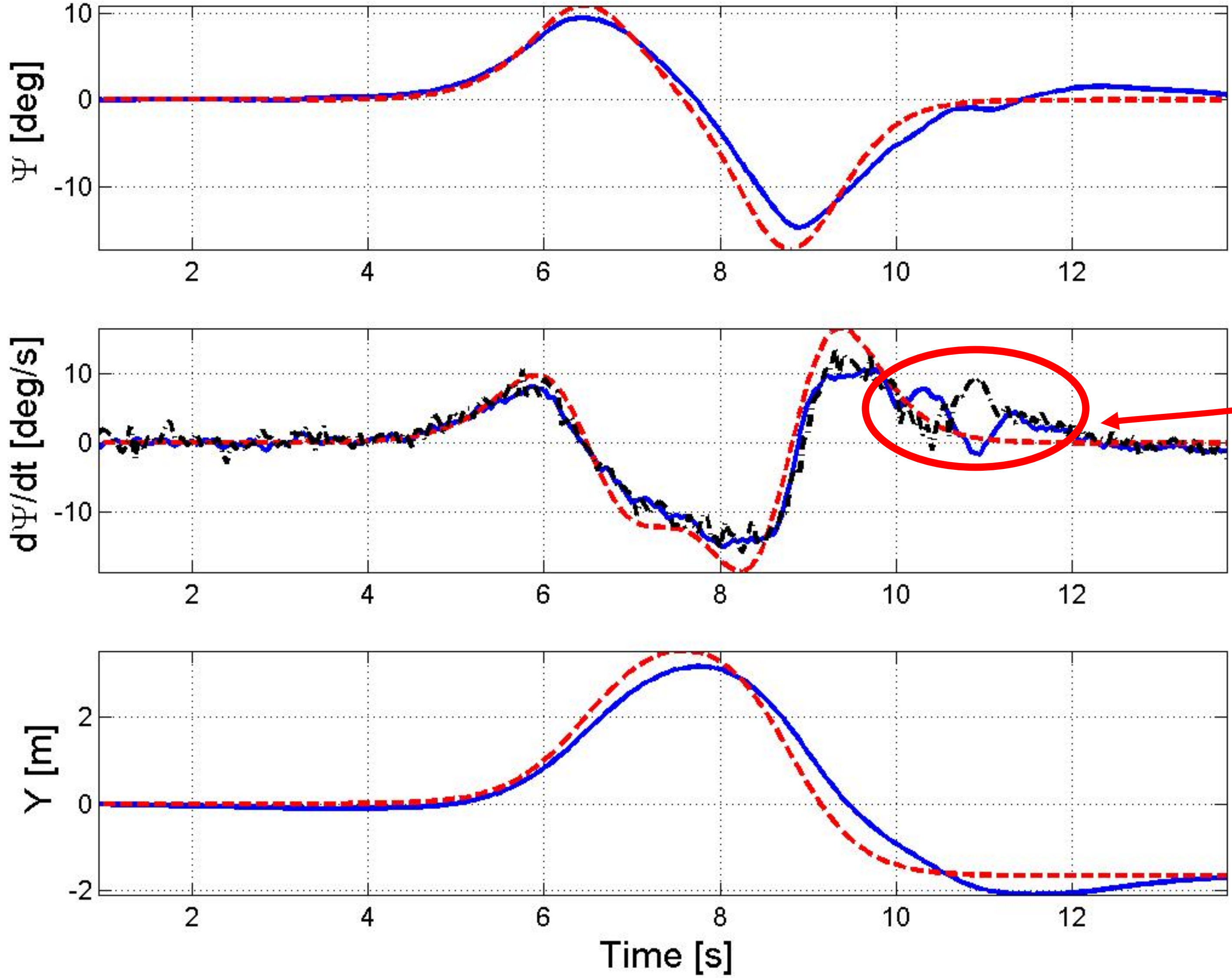
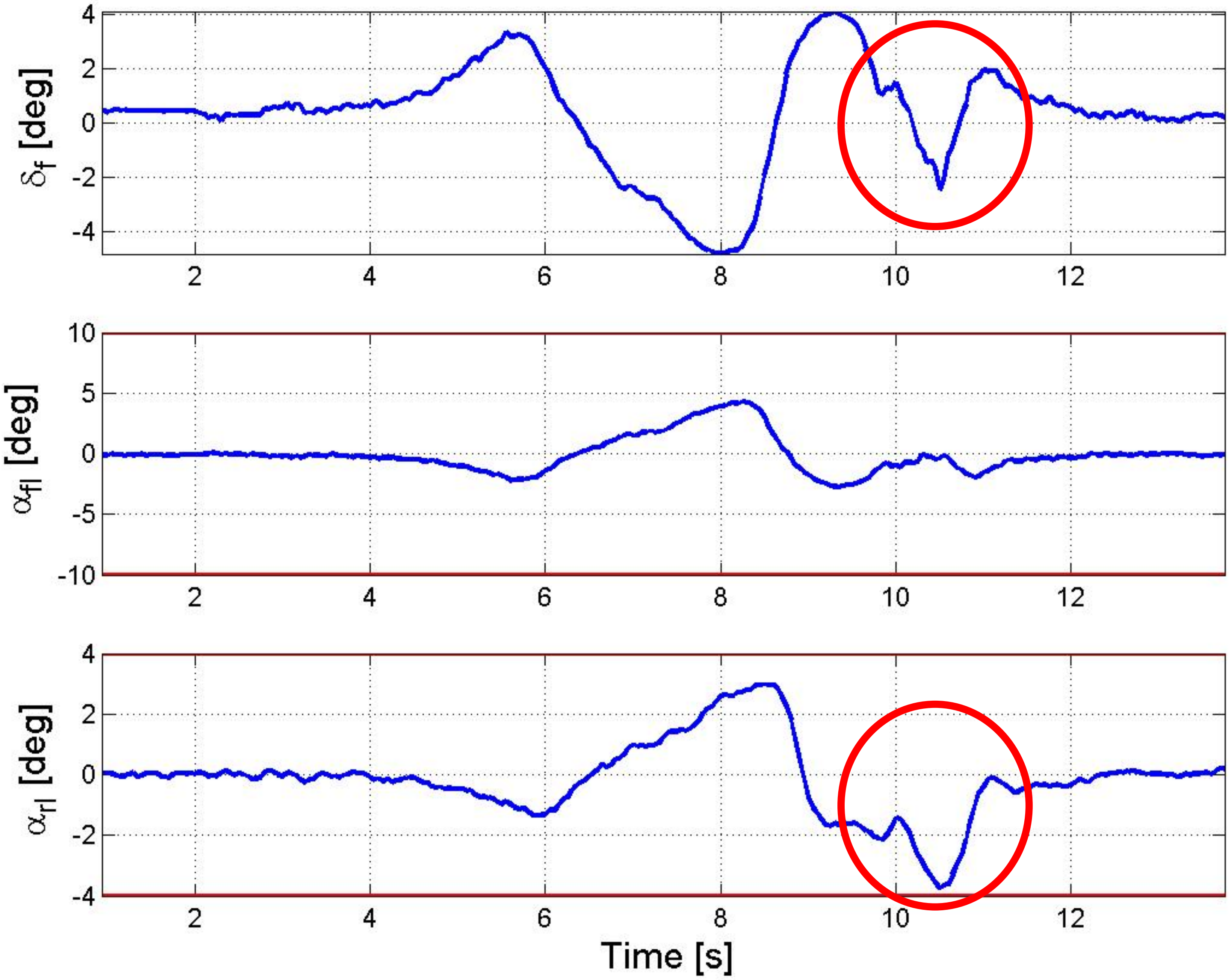
Test @ 40 Kph. Countersteering manoeuvre.

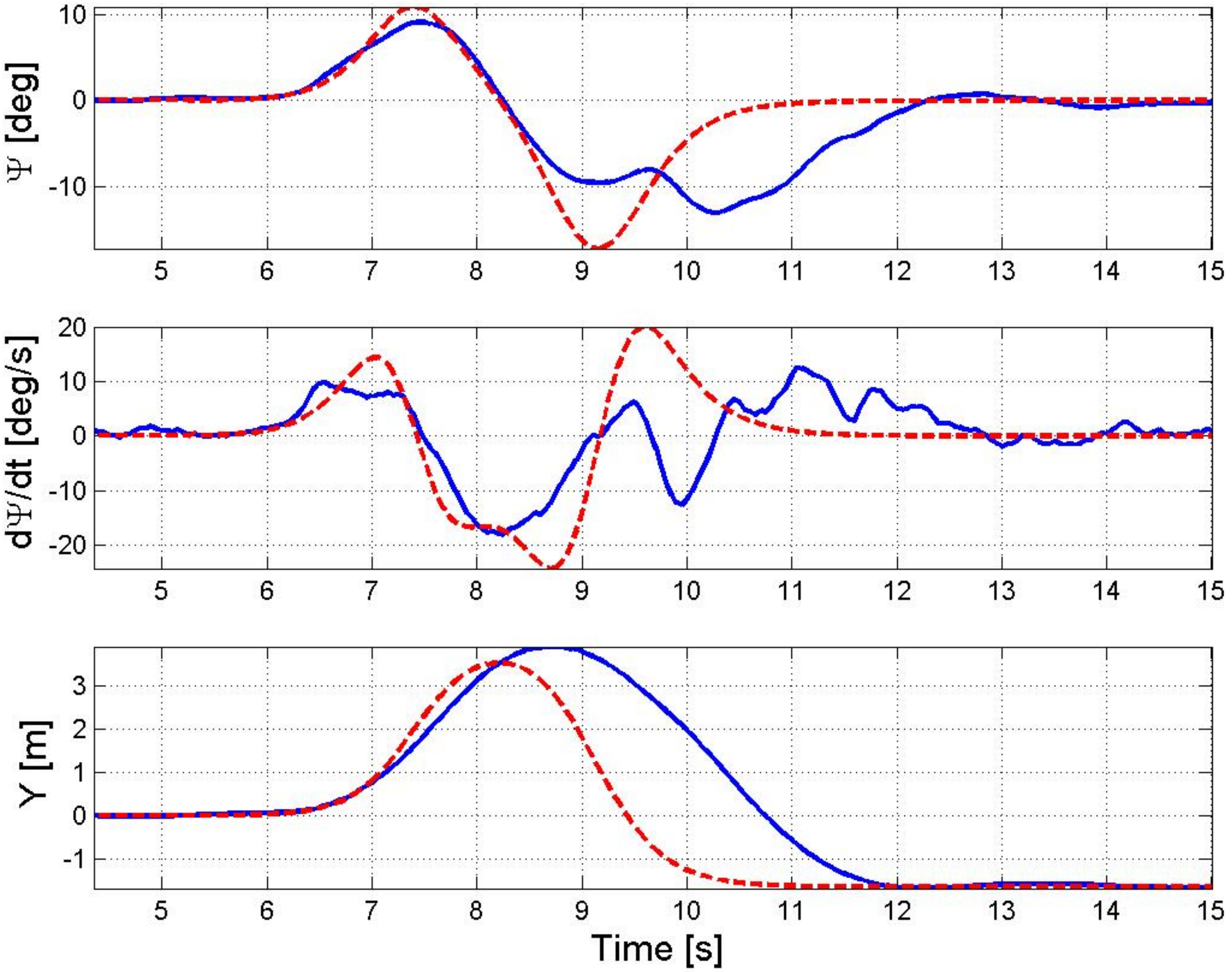


Yaw instability induced by large acceleration

Test @ 40 Kph. Countersteering manoeuvre



Test @ 70 Kph. Countersteering manoeuvre



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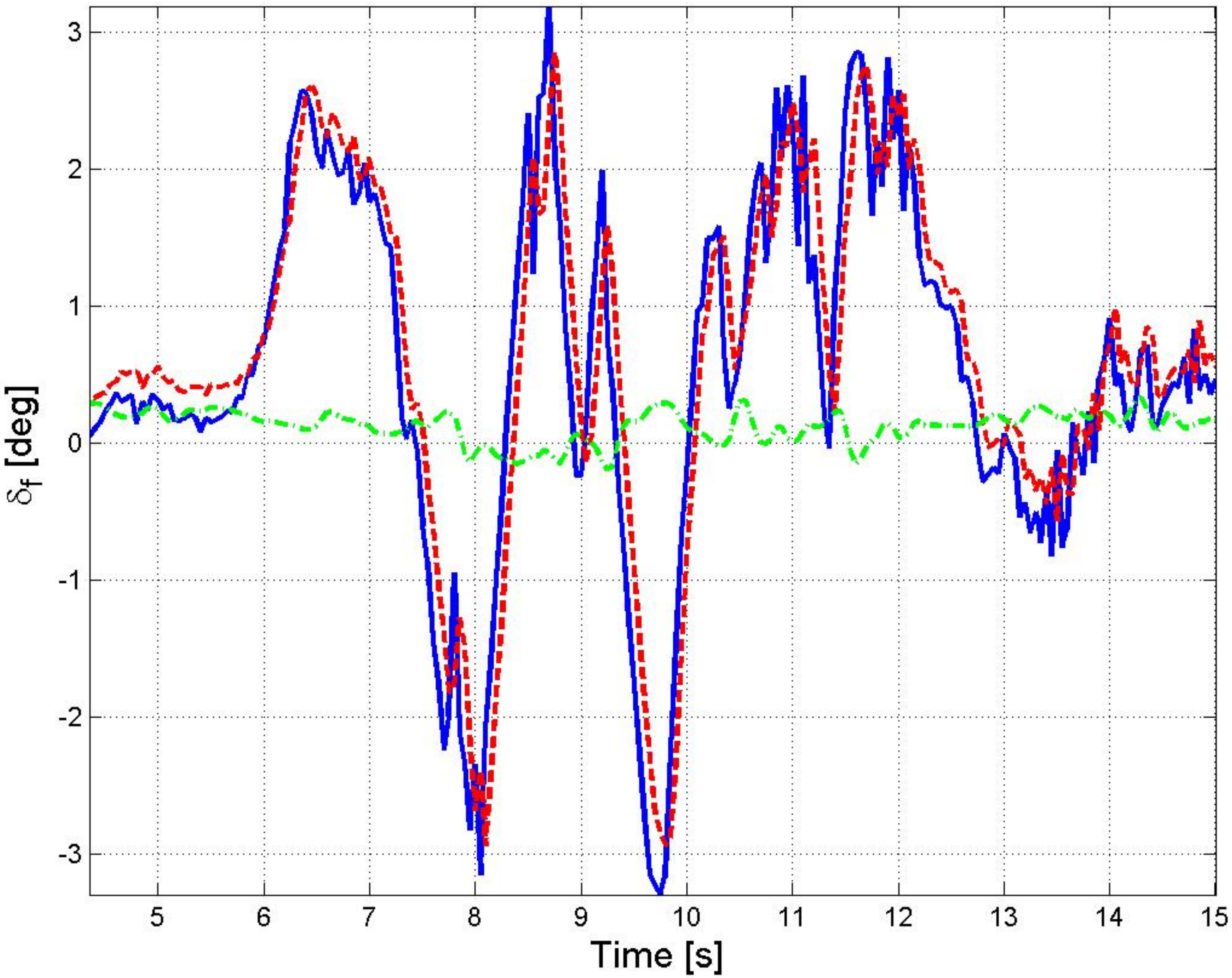

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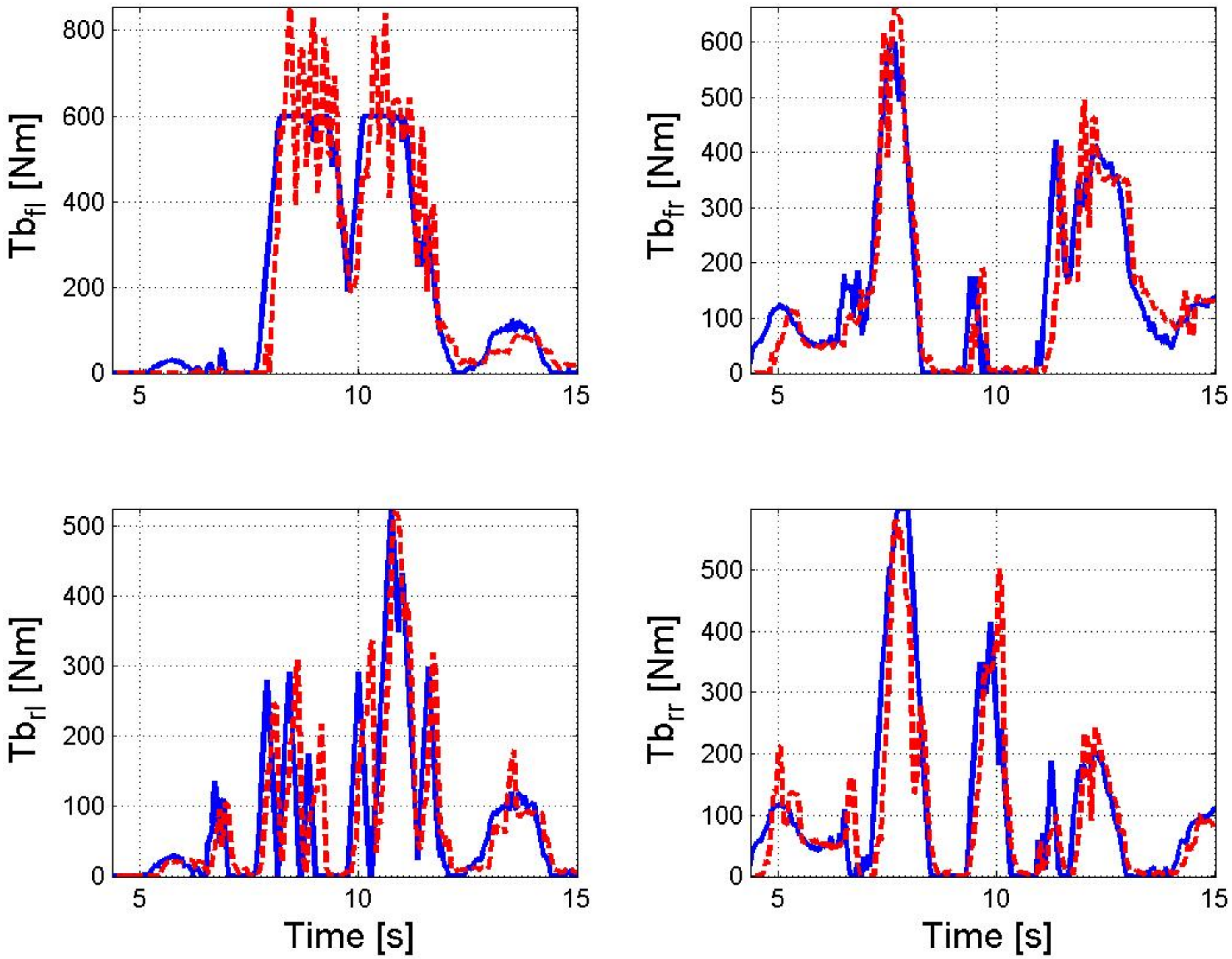


Experimental results

Acknowledgments

- Francesco Borrelli (UC Berkeley)
- Eric Tseng (Ford)
- Davor Hrovat (Ford)

Long Heavy Vehicles Combinations

Long Heavy Vehicles Combinations

Tractor – Semitrailer

Truck – Full Trailer

Truck – Center-axle Trailer

Truck – Dolly – Semitrailer

B-Double

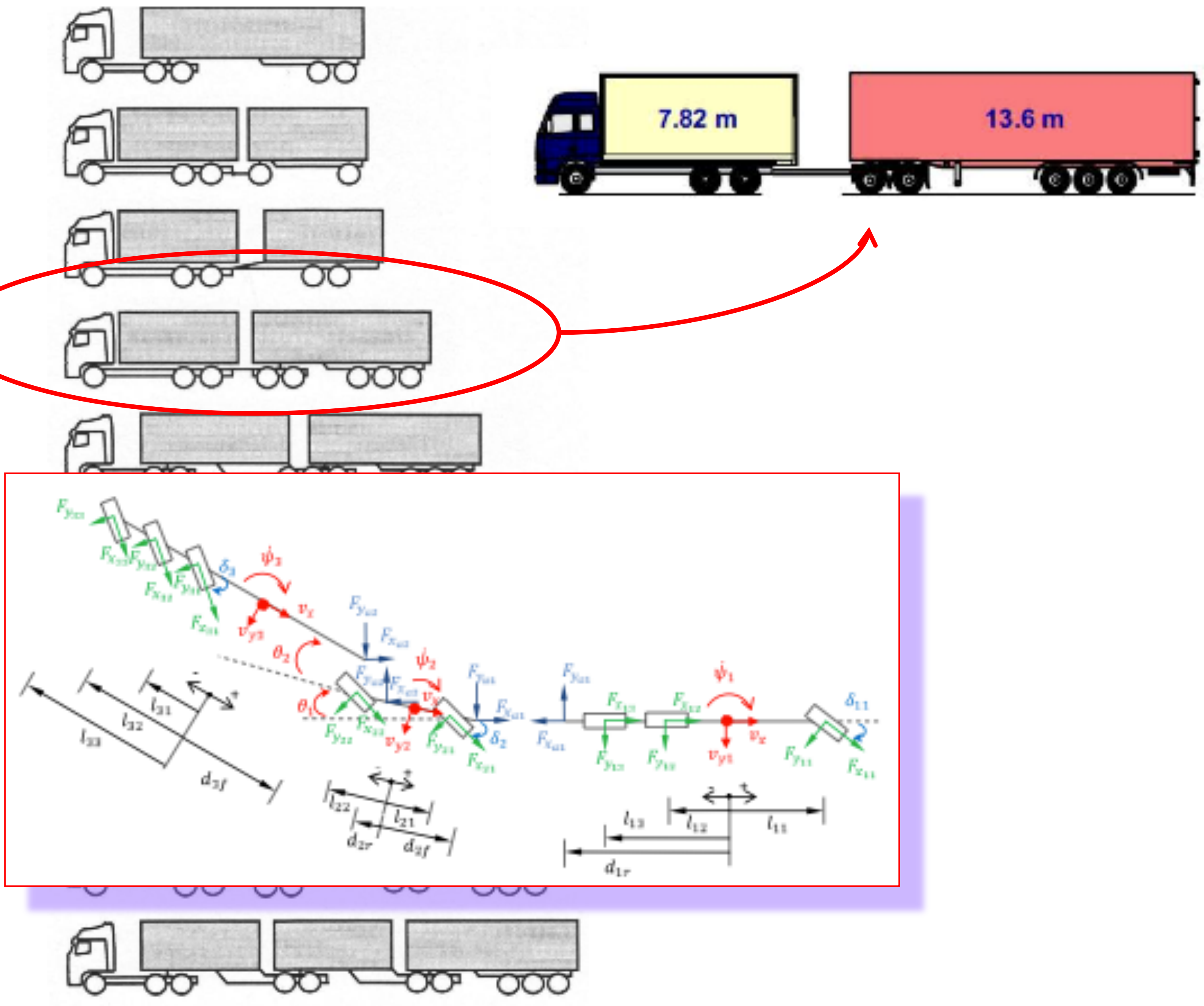
Tractor – Semitrailer – Center-axle Trailer

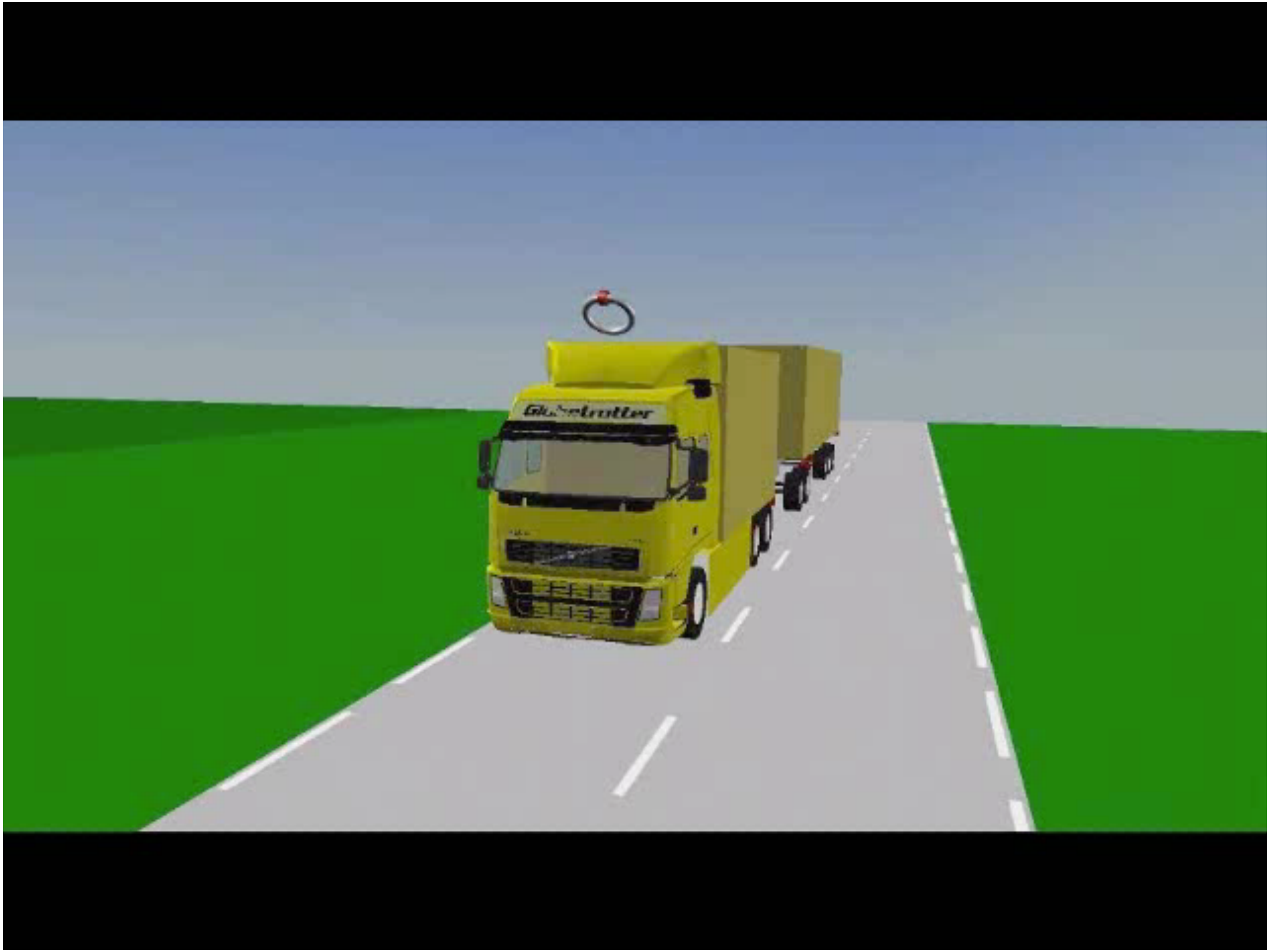
Truck – Double Center-axle Trailer

A-Double

Truck B Double

B-Triple





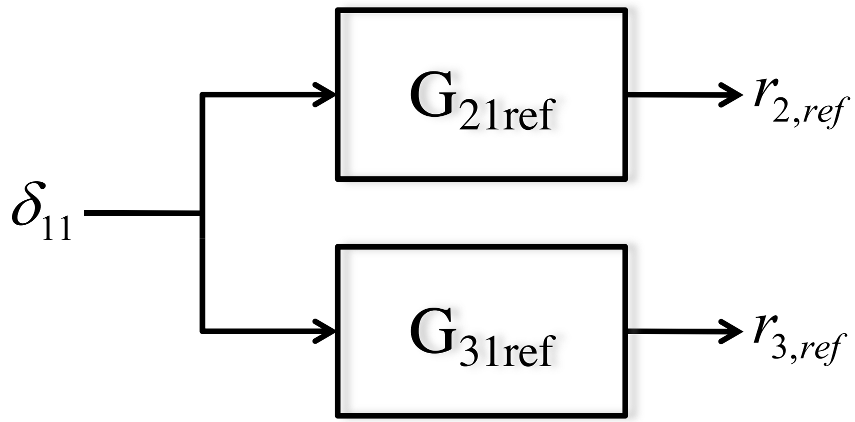
Control Objectives

Reducing the yaw rate rearward amplifications r_2/r_1

and r_3/r_1, by means of the steering angles δ_2, δ_3

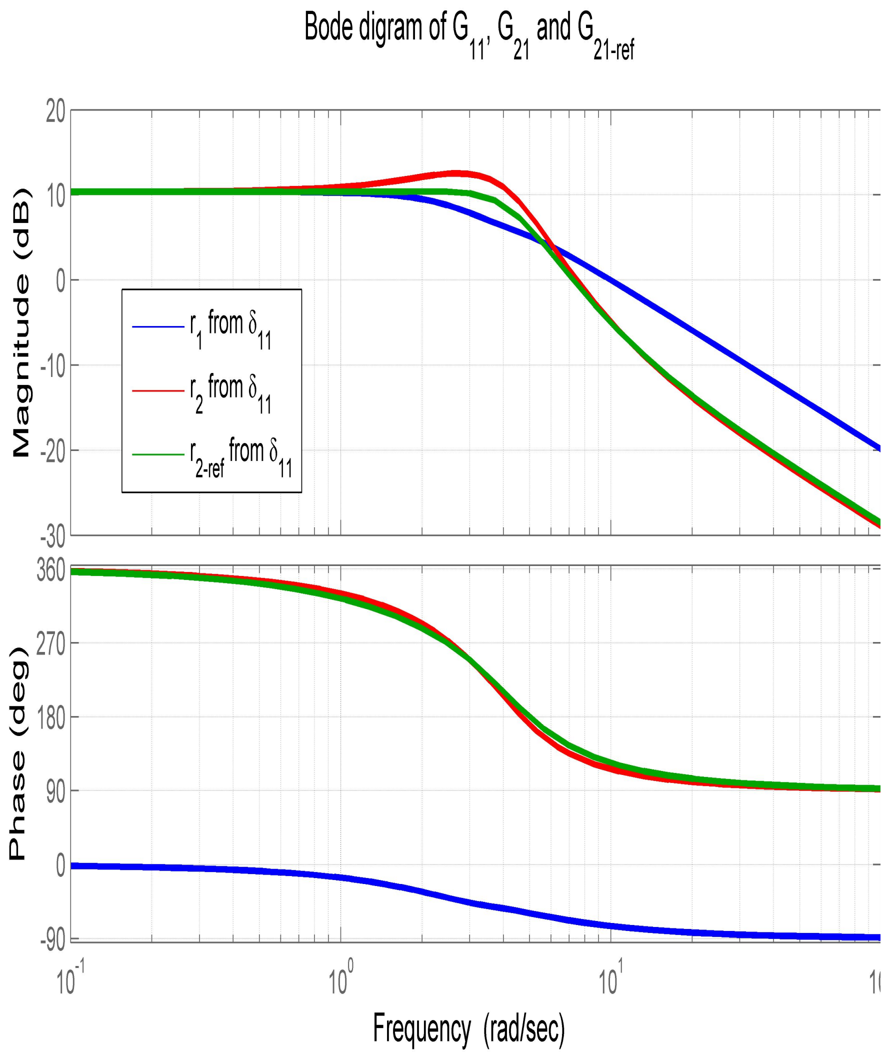
while bounding the steering angles and rates of steering

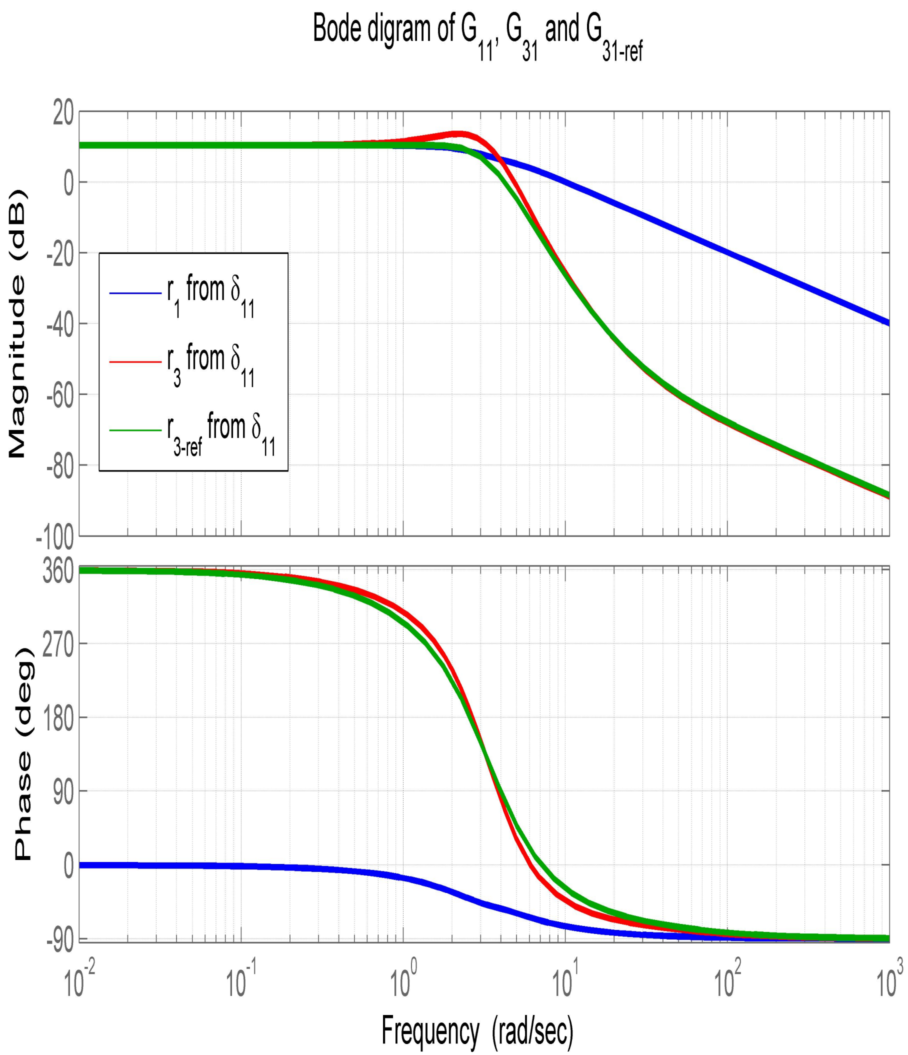
Achieving the control objectives by solving a ***yaw rate tracking problem*** where



Reference models designed in order to remove resonance peaks in the yaw rates responses

Reference Models





MPC Problem Formulation

Vehicle (linear) model

$$x_{k+1} = A(v_x)x_k + B(v_x)u_k + E(v_x)d_k$$
$$y_k = Cx_k$$

$$x = \begin{bmatrix} v_y \\ r_1 \\ \theta_1 \\ \theta_2 \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} \quad u = \begin{bmatrix} \delta_2 \\ \delta_3 \end{bmatrix} \quad d = \delta_{11}$$

$$v_x = \begin{bmatrix} v_x^1 \\ v_x^2 \\ v_x^3 \end{bmatrix} \quad y = \begin{bmatrix} r_2 \\ r_3 \\ \theta_1 \\ \theta_2 \end{bmatrix}$$

Cost function

Yaw rate tracking problem translated into a cost function minimization problem

$$J = \sum_{i=1}^{N} \left(\left\| Q(y_{t+i} - y_{ref,t+i}) \right\|_2^2 + \left\| S u_{t+i-1} \right\|_2^2 + \left\| R \Delta u_{t+i-1} \right\|_2^2 \right)$$

MPC Problem Formulation

$$\min_{\Delta u_t, \ldots, \Delta u_{t+N-1}} \sum_{i=1}^{N} \left(\left\| Q(y_{t+i} - y_{ref,t+i}) \right\|_2^2 + \left\| S u_{t+i-1} \right\|_2^2 + \left\| R \Delta u_{t+i-1} \right\|_2^2 \right)$$

subject to

$$\left. \begin{aligned} x_{k+1} &= A(v_x) x_k + B(v_x) u_k + E(v_x) d_k \\ u_k &= u_{k-1} + \Delta u_k \\ y_k &= C x_k \end{aligned} \right\} \quad \text{Vehicle dynamics}$$

$$\left. \begin{aligned} u_{\min} &\le u_k \le u_{\max} \\ \Delta u_{\min} &\le \Delta u_k \le \Delta u_{\max} \end{aligned} \right\} \quad \text{Actuator limitations}$$

The resulting ***state feedback*** steering control law is

$$u^*(t) = u(t-1) + \Delta u^*(t, x(t))$$

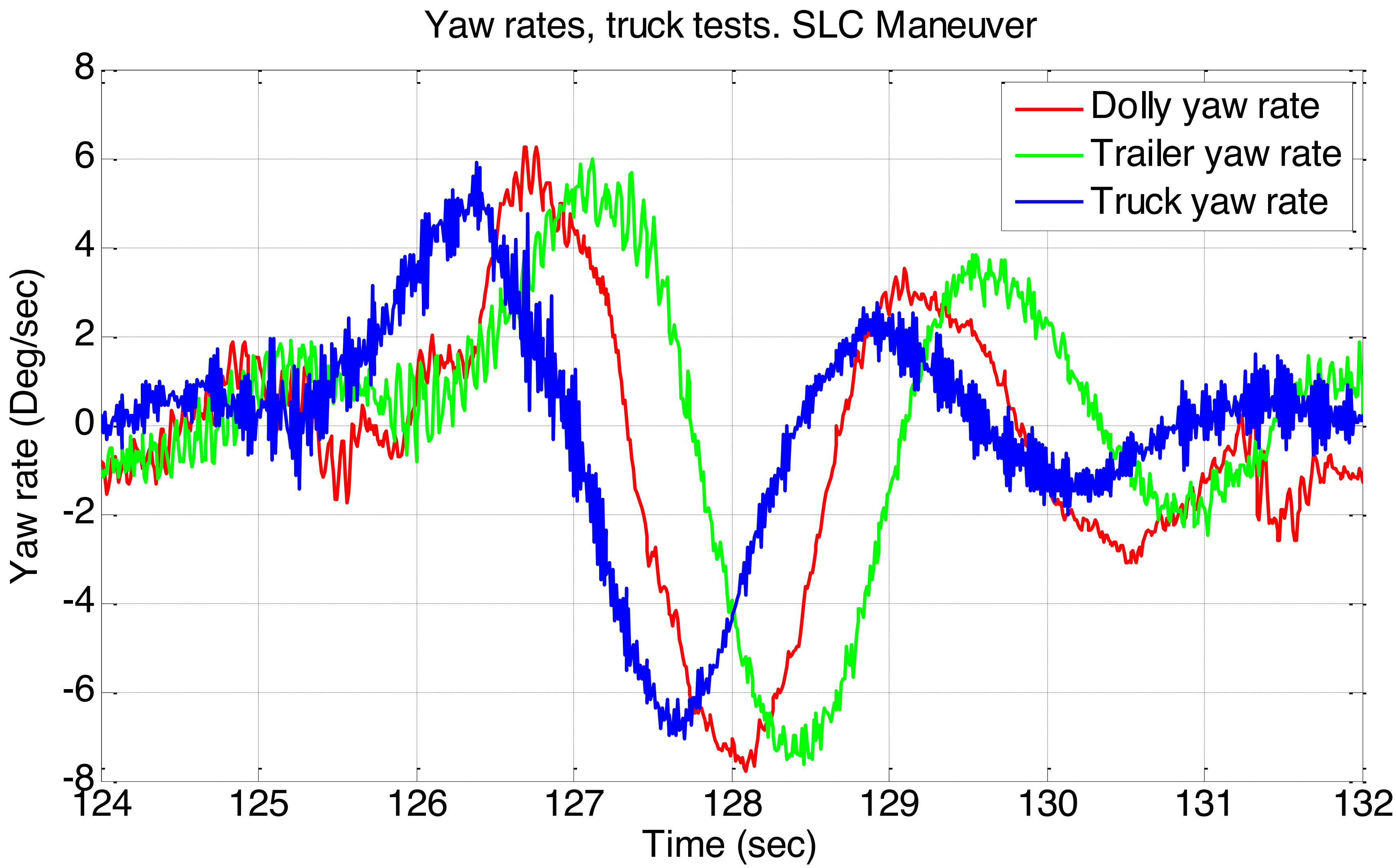
Experimental Results

Testing at Mira Test Center:
Single lane change

Summary

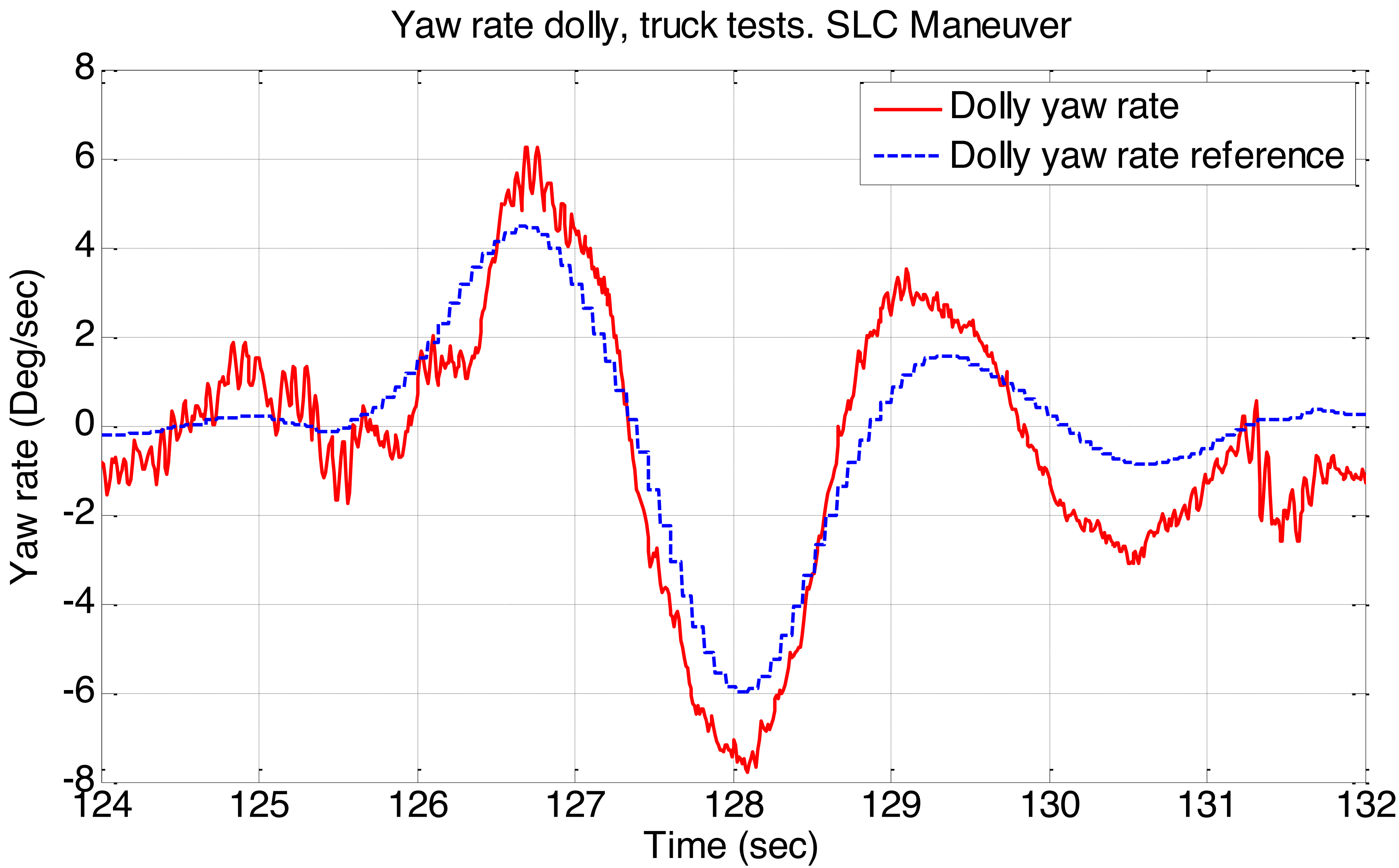
1. RWA close to one
   - Good reference tracking
2. Smooth steering commands
3. Small articulation angles at the end of the maneuver (Both dolly and trailer aligned with the truck)
   - Please note the sensor offset

Yaw Rates

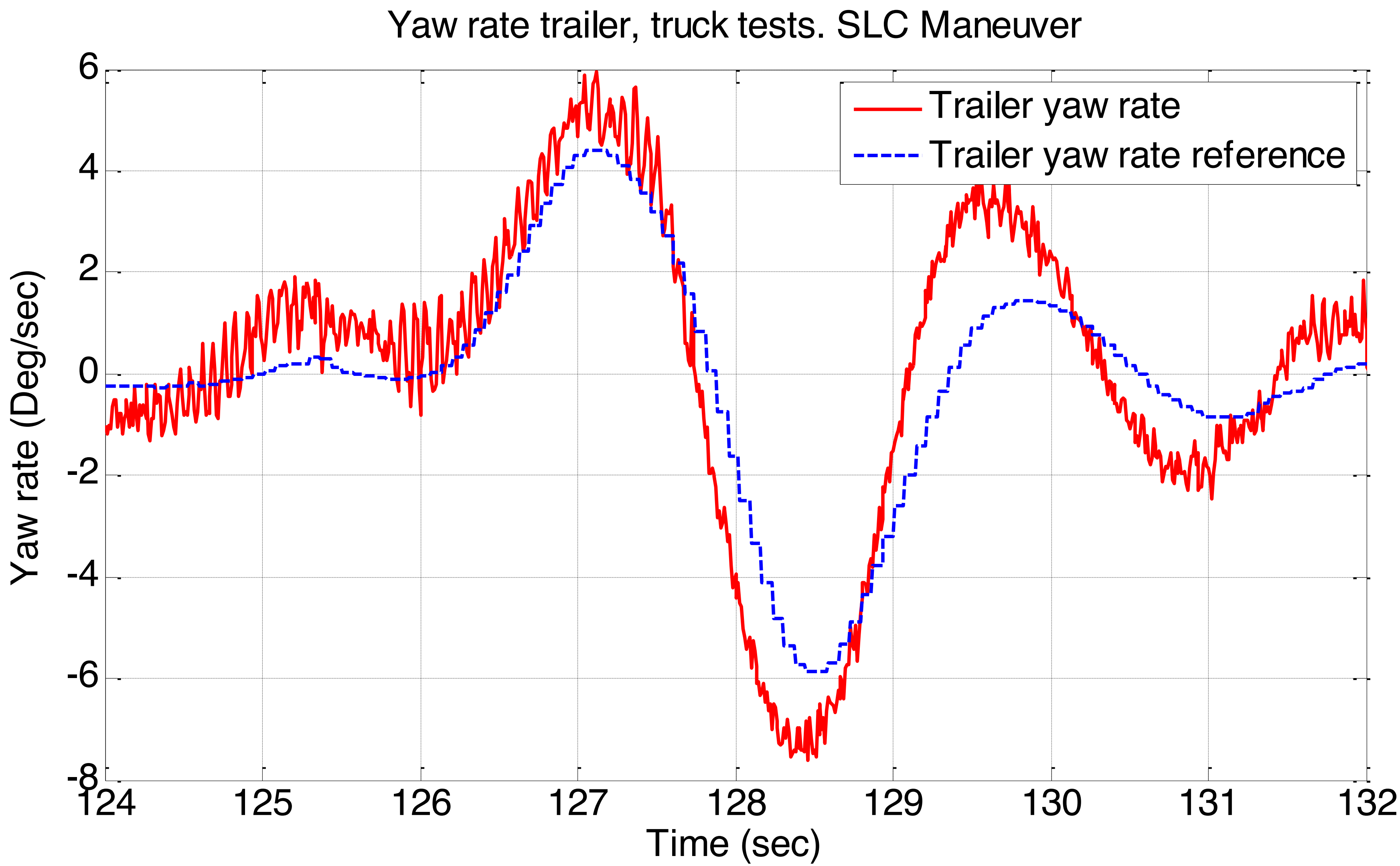


Yaw rates, truck tests. SLC Maneuver
Dolly yaw rate
Trailer yaw rate
Truck yaw rate
Yaw rate (Deg/sec)
Time (sec)
8
6
4
2
0
-2
-4
-6
-8
124
125
126
127
128
129
130
131
132

Dolly Yaw Rate



Trailer Yaw Rate



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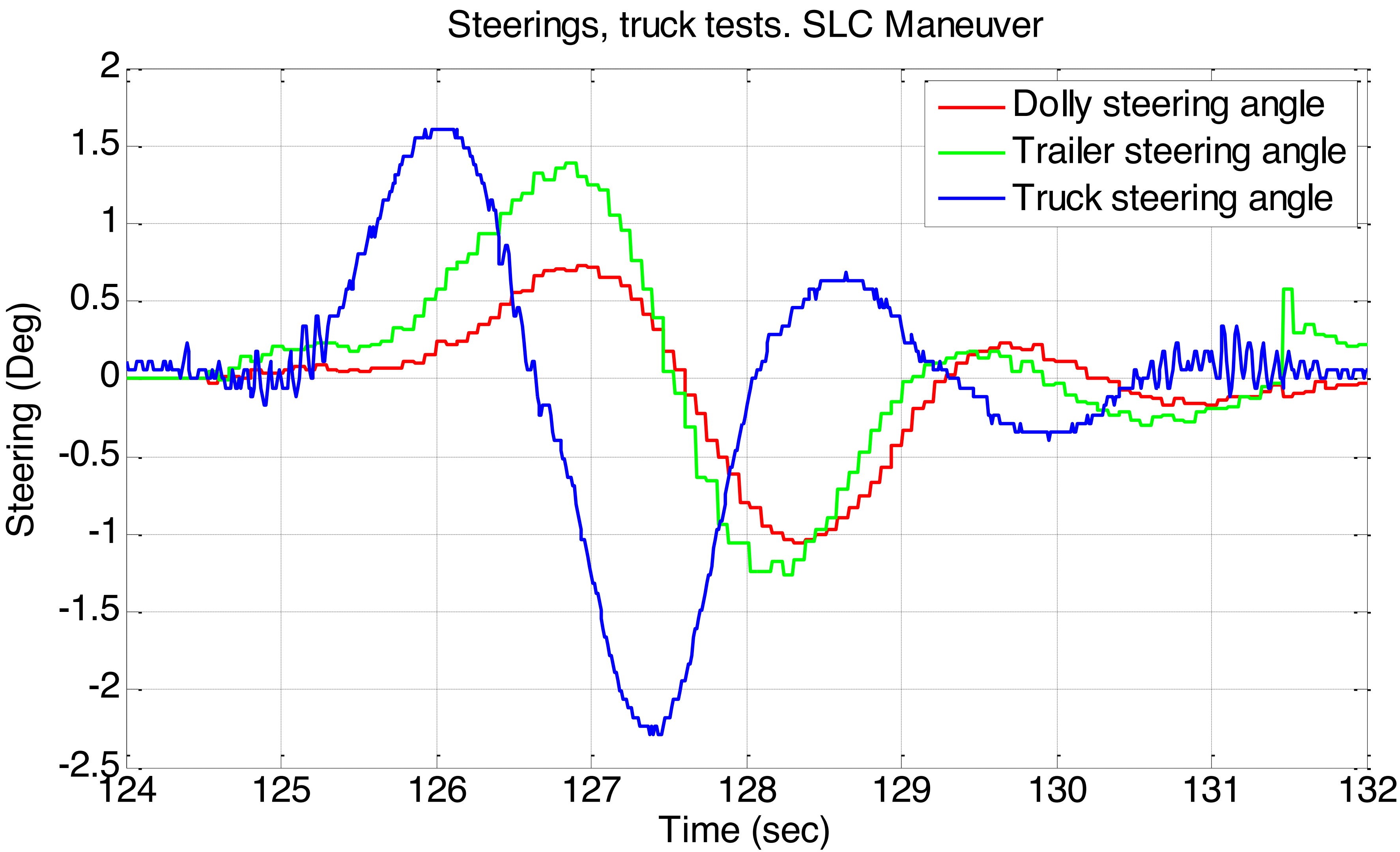
Steering Angles



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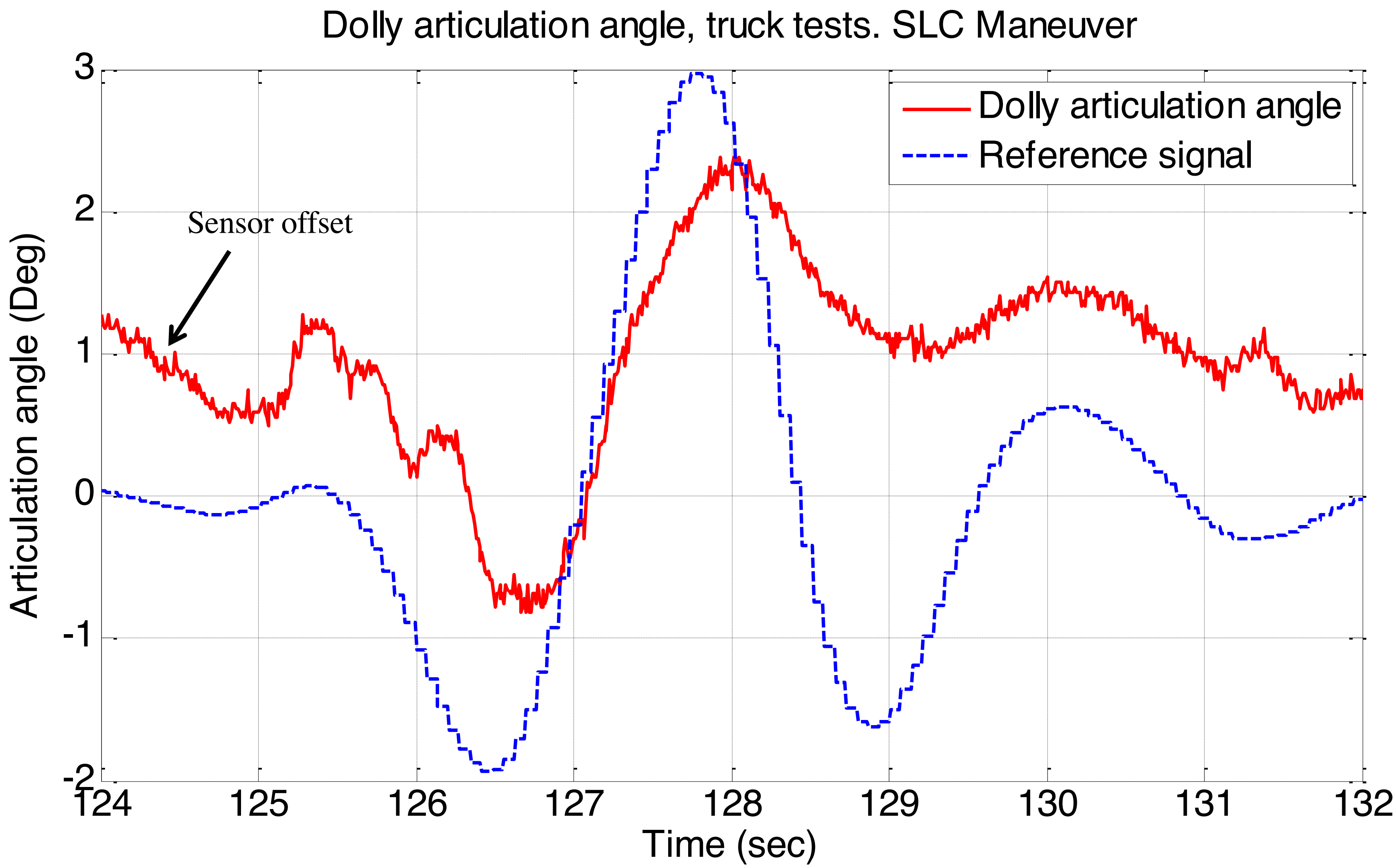
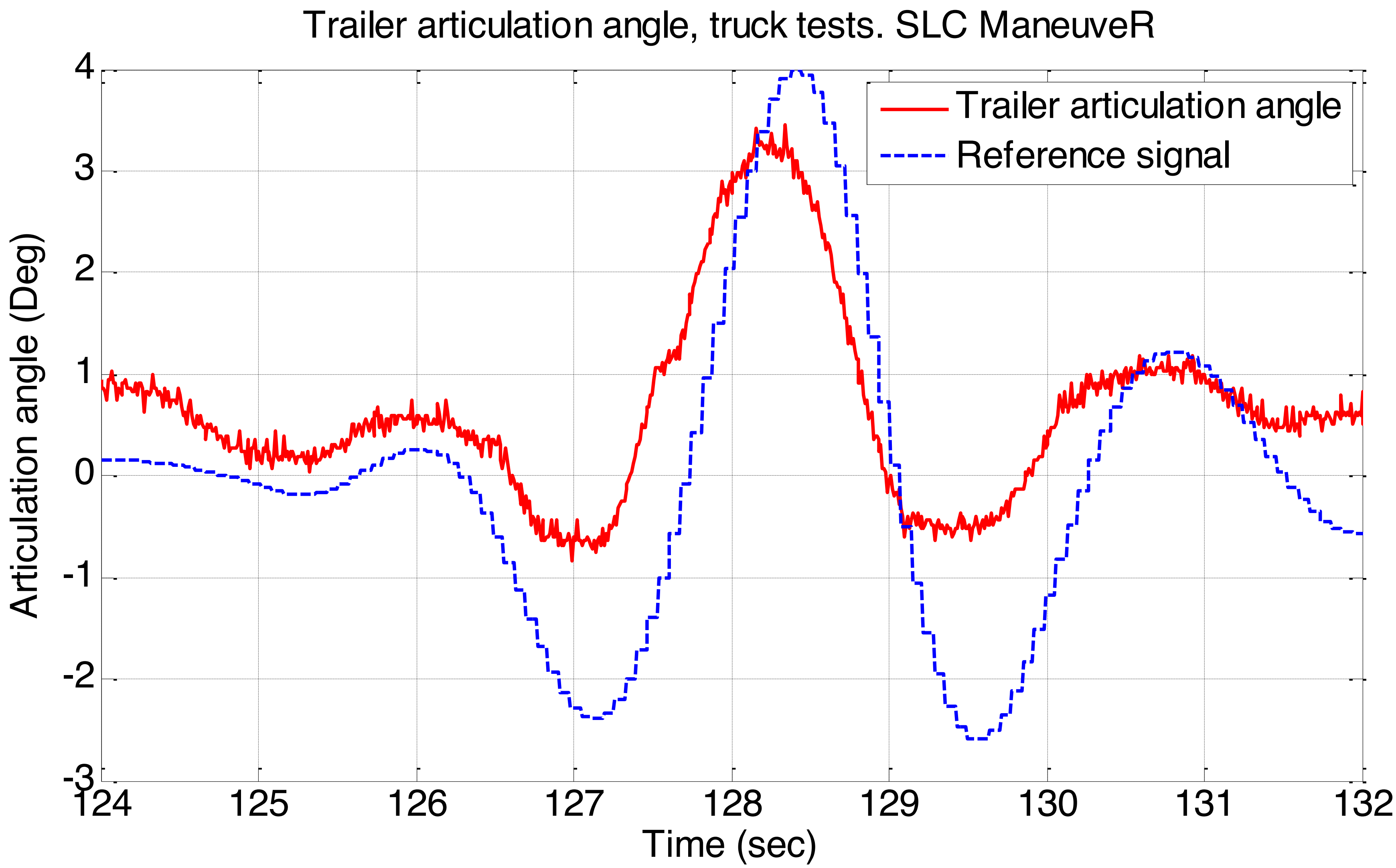
Dolly Articulation Angle



Trailer Articulation Angle




Yaw Rate Control of Dolly and Trailer
Model Predictive Control

Remarks

- Good tracking with ***minimal design and tuning efforts***
- ***Actuators physical limits*** included in control design
- Possibility of easily
  - Include constraints to ***guarantee RWA<1***
  - Include constraints to ***limit lateral accelerations***
  - Include constraints to ***limit off-tracking***
  - ***Combine steering and braking*** commands
  - Guarantee ***perfect alignment of the combination*** despite of sensors offsets

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